ASSIGNMENT

1 a Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{dy}{dt}$ = fresh air inflow rate - fresh air outflow rate

fresh air inflow - $600 ft^3/min$

fresh air outflow - Note; amount flowing out of the ~~room~~ is a function of the amount in the room

$$\frac{600}{20000} = 0.03 min$$

i.e 0.03 of $y(t)$ is the outflow $= 0.03y ft^3/min$

Now;

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be separated and integrated;

$$\frac{dy}{(y-20000)} = -0.03dt$$

Find the integral of both sides

$$\ln(y - 20000) = -0.03t + c$$

$$y - 20000 = e^{(-0.03t + c)}$$

$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

Recall $c = e^c$ = initial equation

$$\therefore y - 20000 = e^{-0.03t} \cdot c \;\ldots\ldots\; (1)$$

At $t=0$, $y(0)=0$ since the room contained no fresh air initially

Put $y=0$, $t=0$ in eqn (1)

$$y - 20000 = e^{(0.03 \cdot 0)} \cdot c$$

$$0 - 20000 = e^{0} \cdot c$$

$$0 - 20000 = 1(c)$$

$$c = -20000 \;\ldots\ldots\; (2)$$

Put eqn (2) in eqn (1)

$$y = 20000 - 20000e^{-0.03t}$$

$y = 20000(1 - e^{-0.03t})$ ...... (1)

Equation (1) above is the model for the amount of fresh air in the room

b $90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20,000$; ie 90% of air in the room

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$= \frac{-2.303}{-0.03}$

$= 76.77 mins$

$\simeq 77 mins$

c $t = 6hrs$

$= 6 \times 60$

$= 360 mins$

Soln

Command window

Clear all

Clc

Close all

Syms y, t

y = 20000 * (1-exp(-0.03* t))

t = 0.5 : 360

Yn = subs(y)

Plot (t, yn)

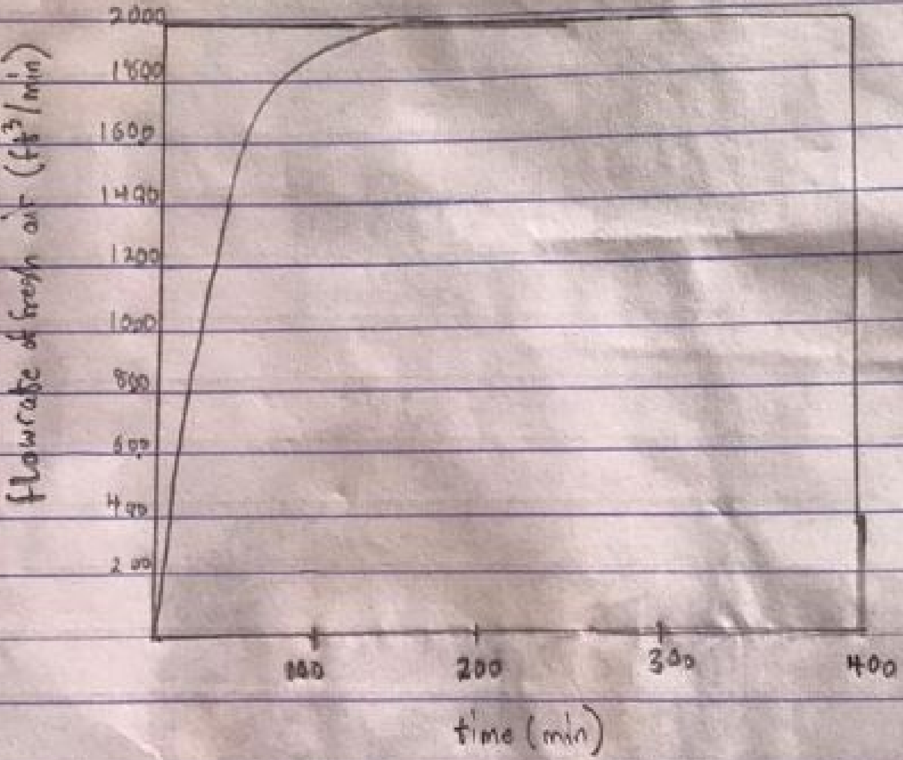
X label ('Time (min)')

Y label ('Flowrate of fresh air (ft^3 (min)')

Grid on

Grid minor

Axis tight



d The steady-state value is 20000ft³ at 215mins (3hr and 35mins) of exponential approach.

e The functions above shows an exponential approach to the limit of 20,000 ft³ as y increases with time. Also, when the steady state value approaches 20000ft³ at 215minutes and continues till 300mins (6hrs). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.