



ENG284

ASSIGNMENT IV

It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows ~~into~~ a room containing 20000 ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially.

(a) Develop a model for the amount of fresh ~~air~~ ^{air} in the room at any time t .

(b) Calculate the time at which 90% of the air in the room will become fresh.

(c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6 \text{ hr}$ using a step time of 5 min.

(d) Determine the steady-state value of the amount of fresh air in the room and

(e) Comment on the result obtained in (d)

Solution

(a) Let $y(t)$ be the amount of air time t in the room
 $\frac{dy}{dt}$ = air inflow rate - fresh air outflow rate

$$\text{fresh air inflow} = 600 \text{ ft}^3/\text{min}$$

$$\text{fresh air outflow} = \frac{600}{20000} = 0.03 \text{ min}$$

$$\text{i.e. } 0.03 \text{ of } y(t) \text{ is the outflow} = 0.03 y \text{ ft}^3$$

$$\frac{dy}{dt} = 600 - 0.03y$$

dt

$$= -0.03y + 600$$

factorize

$$= -0.03(y - 20000)$$

Solve by separating the variables

$$\frac{dy}{dt} = 0.03dt$$

$$y - 20000$$

Integrate both sides

$$\ln(y - 20000) = 0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

$$y - 20000 = e^{-0.03t} e^C$$

$$\text{Recall } C = e^C$$

$$y - 20000 = e^{-0.03t} C \quad \text{--- (1)}$$

$$\text{At } t=0, y(t)=0$$

$$y - 20000 = C e^{-0.03t}$$

$$0 - 20000 = C$$

$$C = -20000 \quad \text{--- (2)}$$

Substitute (2) into (1)

$$y - 20000 = e^{-0.03t} (-20000)$$

$$y = -20000 e^{-0.03t} + 20000$$

$$= 20000 (-e^{-0.03t} + 1)$$

$$\Rightarrow 20000 (1 - e^{-0.03t}) \quad \text{--- (3)}$$

The equation above is the model for the growth of fresh air in the room

$$(b) \quad 90\% = \frac{90}{100} = 0.9$$

$$y = 0.90 \text{ of } 20000$$

$$= 0.9 \times 20000$$

$$= 18000 \text{ ft}^3$$

$$y = 20000 \text{ Recall } y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$e^{-0.03t} = 0.1$$

$$\sim 0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03} = 76.78 \approx 77 \text{ mins}$$

i. The air in the room will be 90% fresh at 27 minutes

(c) Command Window

clear all

clc

close all

Syms Y,t,k

$$Y = 20000 * (1 - \exp(-0.03 * t))$$

$$t = 0:5:360$$

$$Y_n = \text{subs}(Y)$$

$$\text{plot}(t, Y_n)$$

$$\text{xlabel}('TIME (mins)')$$

$$\text{ylabel}('FLOWRATE OF FRESH AIR (FT^3/min)')$$

grid on

grid minor

axis tight

