$y = 20000 = e^{-0.03t} \cdot e^{c}$

$y - 20000 = e^{-0.03t} \cdot y_0$

$y - 20000 = y_0 e^{-0.03t}$

when $t = 0$, $y = 0$

$0 - 20000 = y_0 \cdot e^{-0.03(0)}$

$-20000 = y_0 \cdot 1$

$y_0 = -20000 ft^3/min$

$y - 20000 = -20000 \cdot e^{-0.03t}$

$y = 20000 - (20000 \cdot e^{-0.03t})$

b) 90% of 20 000 cm^3 of fresh air

$= 90/100 \times 20000$

$= 18000 cm^3$ of fresh air

when $y = 18000$ what is t?

$18000 = 20000 - (20000 \cdot e^{-0.03t})$

$18000 - 20000 = -(20000 \cdot e^{-0.03t})$

$-2000 = -(20000 \cdot e^{-0.03t})$

$\frac{-2000}{-20000} = e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$-2.3 = 0.03t$

$t = \frac{-2.3}{-0.03}$

$t = 76.6$ minutes

c) commandwindow

clear

clc

clear all

~~$y = 20000 - (20000 \cdot e^{-0.03}$~~

$y = 20000 - (20000 * \exp(-0.03 * t))$

$t = 0:5:360$

plot (t, y)

xlabel ('time')

ylabel ('volume')

grid on

grid minor

Assignment 4

It is discovered that 600ft³/min of fresh air flows into a room containing 20000ft³ of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially,

a) develop a model for the amount of fresh air in the room at any time, t

b) Calculate the time at which 90% of the air in the room will be room fresh.

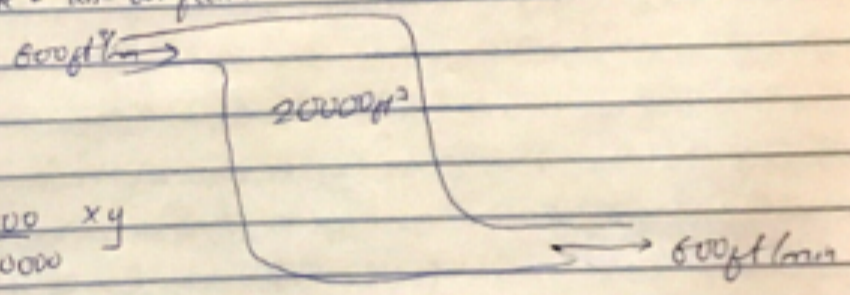
c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for t = 0 to t = 6hr using a step time of 5 min

d) determine the steady-state value of the amount of fresh air in the room, and

e) comment on the result obtained in (d).

Solution

a) $\frac{dy}{dt}$ = air in flowrate - air out flowrate



$\frac{dy}{dt} = y_{in} - y_{out}$

$\frac{dy}{dt} = 600ft^3/min - \frac{600}{20000} \times y$

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03(-20000 + y)$

$\frac{dy}{dt} = -0.03(y - 20000)$

$\frac{dy}{y - 20000} = (-0.03)dt$

Integrating both sides

$\int \frac{1}{y - 20000} dy = \int (-0.05)dt$

$\ln(y - 20000) = -0.03t + c$

Dividing by ln

$(y - 20000) = e^{-0.03t + c}$

d) The steady value is $2 \times 10^9 ft^3$ at 180 mins

e) At time, $t = 180$ mins, the room is filled with $20000 ft^3$ of fresh air and is maintained until the 360 min.