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Initially, no fresh air = 0 ft³ of air

Let $y(t)$ be the amount of fresh air present in ft³

$$\frac{dy}{dt} = A_{r \text{ in}} - A_{r \text{ out}}$$

$$A_{r \text{ in}} = 600 \text{ ft}^3/\text{min}$$

$$A_{r \text{ out}} = \frac{y}{20000} \times 600 \text{ ft}^3/\text{min}$$

$$A_{r \text{ out}} = 0.03 y \text{ ft}^3/\text{min}$$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$\frac{dy}{dt} = -0.03(-20000 + y)$$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\frac{1}{(y-20000)} dy = -0.03 dt$$

Integrating the above equation

$$\int \frac{1}{(y-20000)} dy = \int -0.03 dt$$

$$\ln(y-20000) = -0.03t + C$$

$$y-20000 = e^{-0.03t+C} = e^{-0.03t} \cdot e^C$$

where $e^C = y_0$

$$y - 20000 = y_0 e^{-0.03t}$$

At critical point, $y=0$ and $t=0$

$$0 - 20000 = y_0 e^{-0.03(0)}$$

$$y_0 = -20,000$$

$$y - 20000 = -20000 e^{-0.03t}$$

b) when $y = \frac{90}{100} \times 20000 = 18,000$

$$18000 - 20000 = -20000 e^{-0.03t}$$

$$-2000 = -20000 e^{-0.03t}$$

$$e^{-0.03t} = \frac{-2000}{-20000}$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln 0.1$$

$$t = \frac{-2.3026}{0.03}$$

$$t = 76.75 \approx 77 \text{ min}$$

\Rightarrow 90% of air will become fresh in 77 minutes

c) Command window

clear

clc

close all

t = 0:50:360

y = 20,000 - 20,000 * exp(-0.03*t)

plot(t,y)

xlabel('time (min)')

ylabel('amount (ft³)')

grid on

grid minor

axis tight

