Math assignment 4

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~~Draft~~ (4)

Solution

y = the amount of air at time t in ft^3 in the room.

$\frac{dy}{dt}$ → fresh air inflow rate − fresh air outflow rate.

Fresh air inflow → $600 ft^3/min$

Fresh air outflow

$$= \frac{600}{20000} = 0.03 min.$$

i.e. $0.03y \; ft^3/min$.

$$\frac{dy}{dt} = 600 - 0.03y.$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

$$= \frac{dy}{(y-20000)} = -0.03 dt$$

$$\int \frac{dy}{(y-20000)} = \int -0.03 dt$$

$$= \ln(y-20000) = -0.03t + C.$$

$$y - 20000 = e^{(-0.03t + c)}.$$

$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

recall, $C = e^{c}$

$$y - 20000 = e^{0.03t} \cdot C \quad \text{---- (1)}$$

at $t = 0$, $y(t) = 0$.

$$y - 20000 = e^{-(0.03t)} \cdot C.$$

$$0 - 20000 = e^{0} \cdot C.$$

$$= C = -20000 \quad \text{—— (2)}$$

put eqn 2 into 1

$$y = 20000 - 20000e^{-0.03t}.$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{—— (3)}$$

) Time at which 90% of the air in the room will become fresh.

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000$; is 90% of air in the room

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$-0.03t = \ln(0.1)$

$t = \frac{\ln 0.1}{-0.03}$

$= 76.77 mins \simeq \underline{77 mins}$

c) With the aid of matlab, plot the dynamic response of the amount of fresh air in the room for $t = 0 \rightarrow 6 hrs$.

$t = 6 hrs$

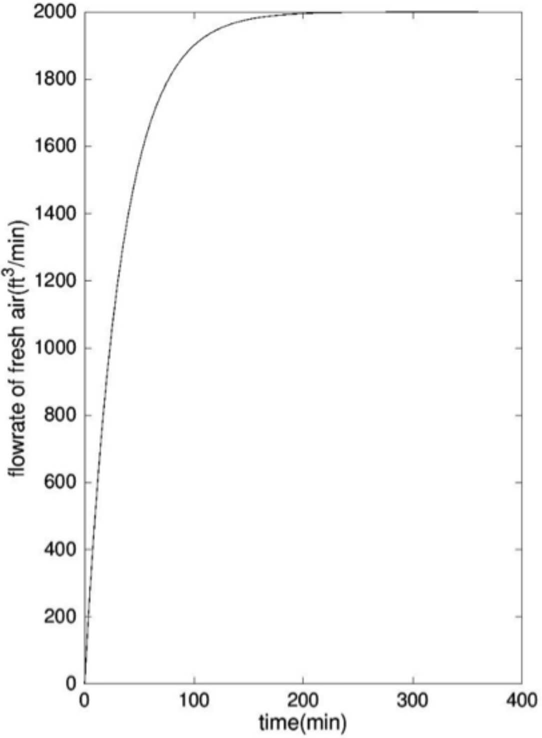
$6 \times 60 s = 360 min$

Soln

Command window

```
Clear
Clc
Close all
Syms y, t
y=20000*(1-exp(-0.03*t))
t=0:5:360
yn=subs(y)
Plot(t,yn)
xlabel('Time(min)')
ylabel('Flowrate of fresh air (ft^3/min)')
Grid on
Grid minor
Axis tight
```

Output



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d) Steady state value:

Steady state value =

20000 ft^3 at 215mins (3hr and 35mins) of exp approach

e) Comment:

The function above shows an exponential approach to the limit of 20000 ft^3 as y increases with time. Also when the SSV approach 20000 ft^3 of 215 mins. The model becomes more realistic in pneumatic technology.