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17/ENG06/038
MECHANICAL ENGINEERING
ENG 282

ASSIGNMENT 4

It is discovered that 600ft³/min of fresh air flows into a room containing 20000 ft³ of air. The mixture which is made practically uniform by circulating fans is exhausted at the rate 600ft³/min. If the room contains no fresh air initially.

a) Develop a model for the amount of fresh air in the room at any time, t.

Answer:

Let $y(t)$ be the amount of air at any t in ft³ in the room.

$\frac{dy}{dt}$ - fresh air inflow rate - fresh air outflow rate.

Fresh air inflow → 600ft³/min

Fresh air outflow → Note: the amount flow out of the room is a function of the amount in the room.

$$\therefore \frac{600}{20000} = 0.03 \text{ min.}$$

i.e 0.03 at $y(t)$ is the outflow rate = $0.03y$ ft³/min.

Now,

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be separated and integrated;

$$\frac{dy}{(y-20000)} = -0.03dt$$

~~This equation~~ Find the integral of both sides

$$\ln(y-20000) = -0.03t + c$$

$$y - 20000 = e^{(-0.03t + c)}$$

$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

Recall $C = e^c$ = initial equation;

$$y - 20000 = e^{-(0.03t)} \cdot C \quad \text{---(1)}$$

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially,

put $y = 0$, $t = 0$ in eqn (1)

$$y - 20000 = e^{-(0.03t)} \cdot C$$

$$0 - 20000 = e^0 \cdot C$$

$$0 - 20000 = C$$

$$C = -20000 \quad \text{---(2)}$$

Put eqn (2) into eqn (1)

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{---(3)}$$

Equation (3) above is the model for the amount of fresh air in the roo

b) Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$y = 0.9 \times 20000$; i.e. 90% of air in the room

$= 18000 ft^3$.

$$y = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln 0.1$$

$$t = \frac{\ln 0.1}{-0.03}$$

$$t = 76.77 \text{mins}$$

$$\simeq 77 \text{mins}$$

With the aid of matlab, plot the dynamic response of the amount of fresh air in the room for t = 0, to t = 6hrs using a step of 5mins

Note: t = 6 hrs

= 6×60s

= 360mins.

Solution

Comment window

```
clear all
clc
close all
syms y,t
y = 20000*(1-exp(-0.03*t)
t = 0:5:360
yn = subs(y)
plot (t,yn)
x label ('Time (min)')
Y Label ('Flowrate of freshair (ft^3/min)'
Grid on
Grid minor
Axis tight
```

Output

Determine the steady-state value of the amount of fresh air in the room

Answer

The steady state value is 20000ft^3 at 215min (3hrs and 35mins) of exponential approach.

Comment on answer in (d)

The steady state value approaches 20000 ft^3 at 215 mins and continues. The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.