Ugah-Uche Chukwunenye
17/ENG04/071

ELECT/ELECT ASSIGNMENT IV

1. It is discovered that 600ft³/min of fresh air flows into a room containing 20000ft³ of air. The mixture which is made practically Uniform by circulating fans, is exhausted at the rate of 600ft³/min. If the room contains no fresh air initially (a) develop a model for the amount of fresh air in the room at any time t.

Answer

Let $y(t)$ be the amount of air at any time t in ft³ in the room

$\frac{dy}{dt} \rightarrow$ fresh air inflow rate − fresh air outflow rate

fresh air Inflow → 600ft³/min

Fresh air outflow → Note:- The amount flowing out of the room is a function of the amount in the room

$\therefore \frac{600}{20000} = 0.03 min$

i.e 0.03 of $y(t)$ is the outflow $= 0.03y$ ft³/min

Now:-

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be separated and integrated;

$$\frac{dy}{(y-20000)} = -0.03dt$$

find the integral of both sides.

$$\ln(y-20000) = -0.03t + c$$

$$y - 20000 = e^{(-0.03t + c)}$$

$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

Recall $c = e^c$ = initial equation.

$$\therefore y - 20000 = e^{-0.03t} \cdot C \quad \text{------ (1)}$$

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially,

put $y = 0$; $t = 0$ in eqn(1)

$$y - 20000 = e^{-(0.03t)} \cdot c$$

$$0 - 20000 = e^{0} \cdot c$$

$$0 - 20000 = 1(c)$$

$$c = -20000 \quad \text{-------- (2)}$$

put eqn (2) in eqn(1)

$\therefore y = 20000 - 20000 e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ------- (3)

Equation (3) above is the model for the amount of fresh air in the

(b) Calculate the time at which 90% of the air in the room will become fresh

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20{,}000$; i.e 90% of air in the room

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$\therefore t = \frac{\ln(0.1)}{-0.03}$

$= \frac{-2.303}{-0.03}$

$= 76.77 mins$

$\simeq 77 mins$

c With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for t = 0 to t = 6hrs, using a step of 5min.

Note: $t = 6hrs$

$= 6 \times 60s$

$= 360 mins$

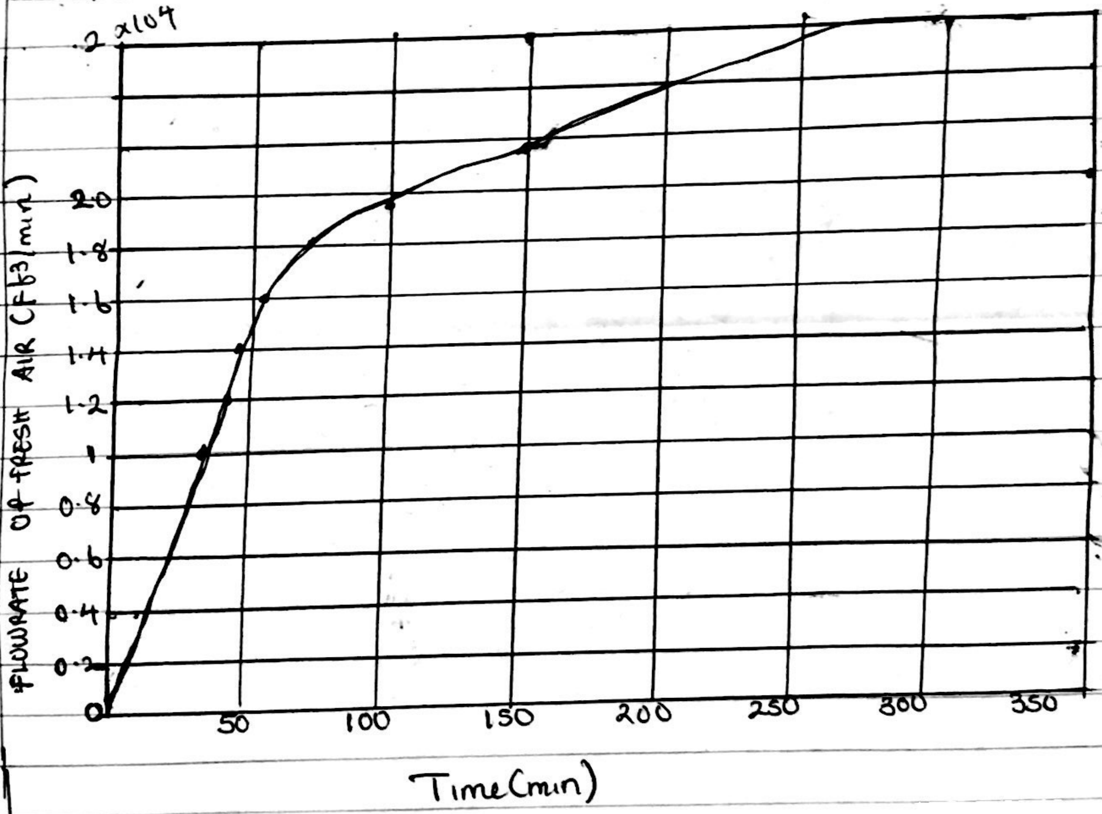
Solution

Command window

```
Clear all
clc
Close all
syms y, t
y = 20000 * (1-exp(-0.03* t))
```

```
t= 0:5:360
yn = subs(y)
plot(t,yn)
xlabel('Time(min)")
ylabel('flowrate of fresh air (ft^3/min)')
Grid on
Grid minor
Axis tight.
```

<u>Output</u>



(d) Determine the steady-state value of the amount of fresh air in the room.

The steady-state value is 20000 ft^3 at 215 mins (3hr and 35 mins) at exponential approach

e. Comment on answer in (d)

The functions above shows an exponential approach to the limit of 20000ft³ as y increases with time. Also, when the steady state value approaches 20000ft³ at 215minutes and continues till 300mins (6hrs). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.