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Assignment

It is discovered that $600 ft^3/min$ of fresh air flows into a room containing $20000 ft^3$ of air. The mixture which is made practically uniform by circulating fans, is exhausted at a rate of $600 ft^3/min$. If the room contains no fresh air initially

a) develop a model for the amount of fresh air in the room at any time, t.

Answer

let $y(t)$ be the amount of air at time t in (ft^3) in the room

$$\frac{dy}{dt} = \text{fresh air inflow rate} - \text{fresh air outflow rate}$$

fresh air inflow => $600 ft^3/min$

fresh air outflow => N/B :- the amount flowing out of the room is a function of the amount in the room

$$\therefore \frac{600}{20000} = 0.03 min$$

i.e 0.03 of $y(t)$ is the outflow = $0.03 y ft^3$.

Now;

$$\frac{dy}{dt} = 600 - 0.03y$$

$= -0.03y + 600$

$= -0.03(y - 20000)$

Thus the equation is seperable and can be solved.

$$\frac{dy}{(y - 20000)} = -0.03\,dt$$

Integrate both sides.

$$\ln(y - 20000) = -0.03t + c$$

$$y - 20000 = e^{-0.03t + c}$$

$$y - 20000 = e^{-0.03t}\, e^{c}$$

Recall $C = e^{c}$ = initial Condition

$$y = 20000 = e^{-0.03t}\, C \quad \text{——— (*)}$$

At $t = 0$, $y(t) = 0$ Since the room no fresh air initially

$$y - 20000 = C\, e^{-0.03(t)}$$

$$0 - 20000 = C$$

$$C = -20000 \quad \text{——— (a)}$$

Put (a) in equation (*)

$$y = 20000 - 20000\, e^{-0.03t}$$

$$y = 20000\,(1 - e^{-0.03t}) \quad \text{——— (**)}$$

The equation above is the model for the amount of fresh air in the room.

Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100} = 0.9$$

$y = 0.9$ of 20000 i.e 90% of air in the room

$= 0.9 \times 20000$

$= 18\,000\ ft^3$

$\therefore \quad y = 20000\left(1 - e^{-0.03t}\right)$

$18000 = 20000\left(1 - e^{-0.03t}\right)$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$= 0.1$

$-0.03t = \ln(0.1)$

$t = \dfrac{\ln(0.1)}{-0.03}$

$t = \dfrac{-2.30}{-0.03} = 76.77 = 77\ mins$

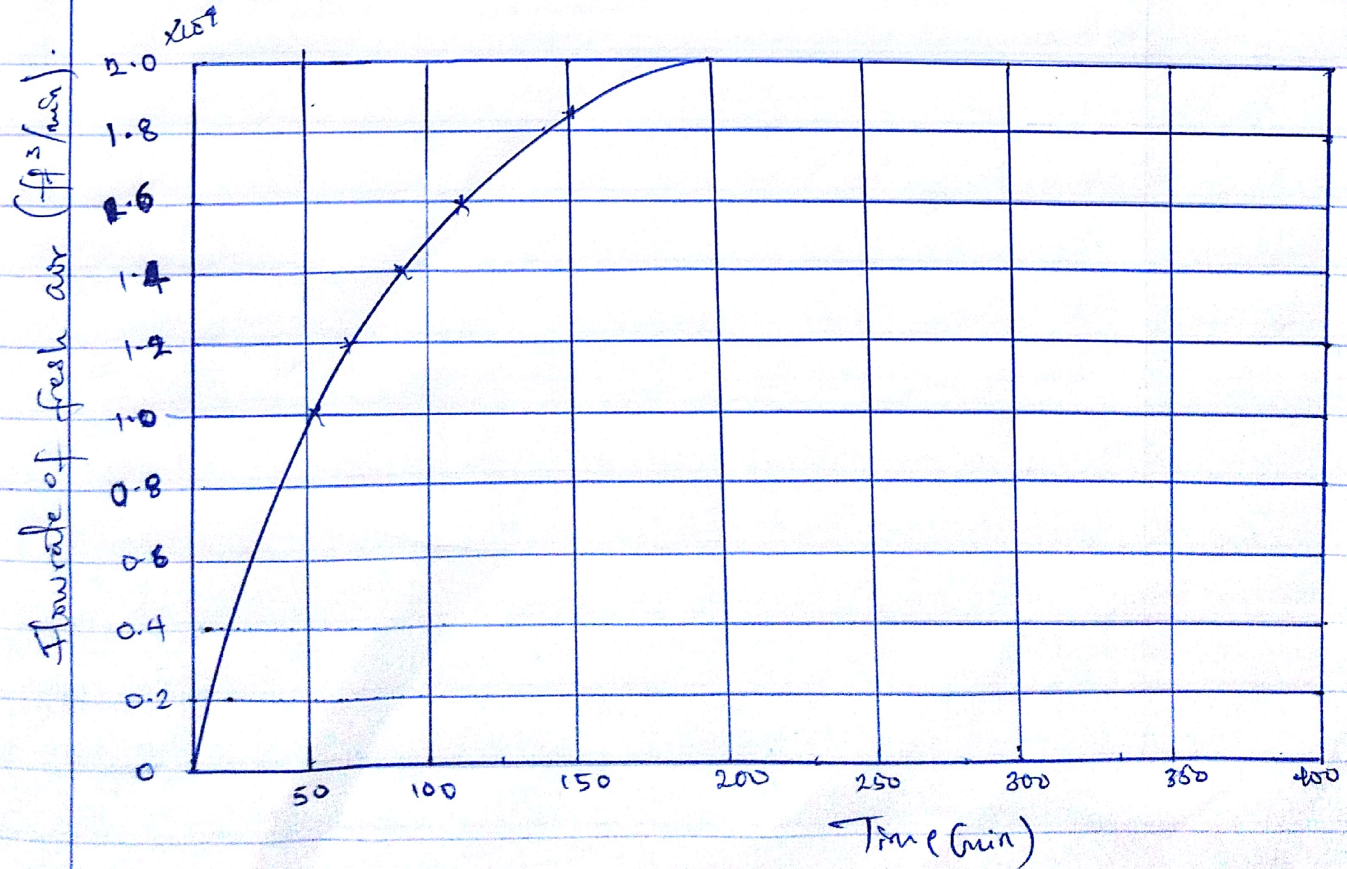
$\therefore$ the air in the room will be 90% fresh at 77 minutes

(c) With the aid of MATLAB, Plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6hr$ with a step of 3min

N/B $\div$ $t = 6hrs = 6 \times 60 = 360$ minutes.

Codes

```
Command window
clear all
clc
close all
Syms y, t, k
Y = 20000*(1-exp(-0.03*t))
  t = 0:5:360
  Yn = Subs(y)
  plot (t, Yn)
  xlabel ('TIME(min)')
  ylabel ('FLOW RATE OF FRESH AIR (Ft^3/min)')
  grid on
  grid minor
  axis tight
```



d) Determine the steady-state value of the amount of fresh air in the room.

Answer

The steady-state value is 20000 ft^3 at 215 mins (3 hours 33 minutes) of exponential approach.

e) Comment on answer in (d)

The function shows an exponential approach to the limit of 20000 ft^3 as g increases with time. Also when the steady state value approach 20000 ft^3 at 45 mins and continues till 360 minute (6 hours). the model discussed becomes more realistic in pneumatic technology although ; maybe difficult because mixing may be imperfect.