

17/ENG01/021

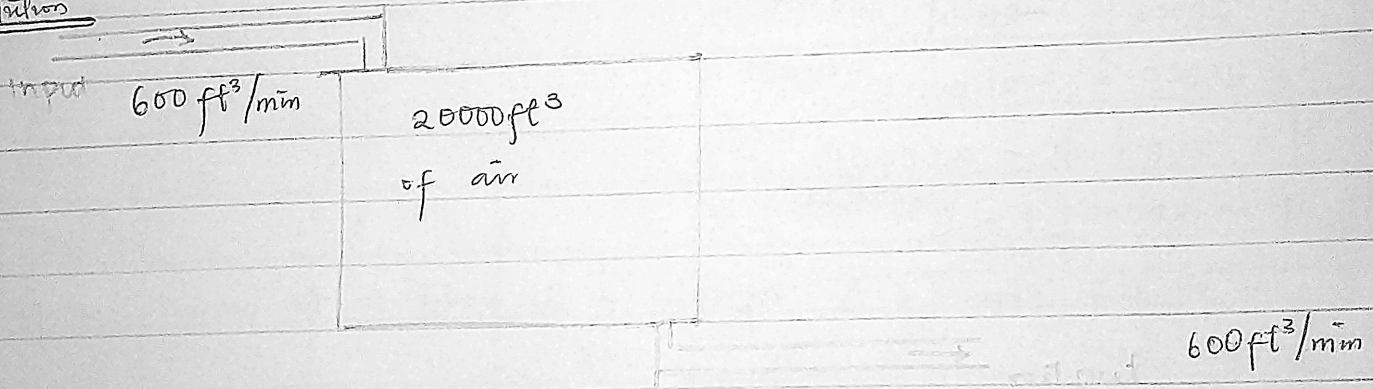
CHEMICAL ENGINEERING

ASSIGNMENT 4 ENA 282

It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows into a room containing 20000 ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (ft^3/m). If the room contains no fresh air initially,

- (a) develop a model for the amount of fresh air in the room at any time t ,
- (b) Calculate the time at which 90% of the air in the room will become fresh
- (c) with the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6 \text{ hr}$ using a step time of 5 min .
- (d) determine the steady state value of the amount of fresh air in the room
- (e) Comment on the result obtained in (d).

Solution



let y be the amount of air at time t in (ft^3) in the room

$$\frac{dy}{dt} = \text{air in flow rate} - \text{fresh air out flow rate}$$

fresh air in flow \rightarrow ~~$600 \text{ ft}^3/\text{min}$~~ $600 \text{ ft}^3/\text{min}$

fresh air out flow rate $\rightarrow \frac{600}{20000} = 0.03 \text{ min}$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$dy \frac{1}{(y - 20000)} = -0.03 dt$$

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

$$y - 20000 = e^{-0.03t} \cdot e^C$$

where e^C is initial condition y_0

$$y - 20000 = e^{-0.03t} y_0$$

$$y - 20000 = y_0 e^{-0.03t}$$

at $t=0$, $y=0$

$$y - 20000 = y_0 e^{-0.03t}$$

$$0 - 20000 = y_0 e^{-0.03(0)}$$

$$-20000 = y_0$$

substituting $y_0 = -20000$ into the initial equation

$$y - 20000 = -20000 e^{-0.03t}$$

$$y - 20000 = -20000 e^{-0.03t}$$

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t})$$

The equation above is the equation ^{of} the model for the amount of fresh air in the room

$$\textcircled{b} 90\% = 90/100 = 0.9$$

$$y = 0.9 \text{ of } 20000$$

$$= 0.9 \times 20000 = 18000 \text{ ft}^3$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln 0.1$$

$$t = \frac{\ln(0.1)}{-0.03} = 76.75 \text{ mins.}$$

command window

clear

clc

close all

t = 0:5:360

$$y = 20000 * (1 - \exp(-0.003 * t))$$

Yn = subs(y)

plot(t, Yn)

xlabel('Time (min)')

ylabel('Flourate of fresh air')

grid on

grid minor

axis tight

output
↑
fresh air

2.8

2.6

2.4

2.2

2.1

2.0

1.8

1.6

1.4

1.2

1.0

0.8

0.6

0.4

0.2

0

50

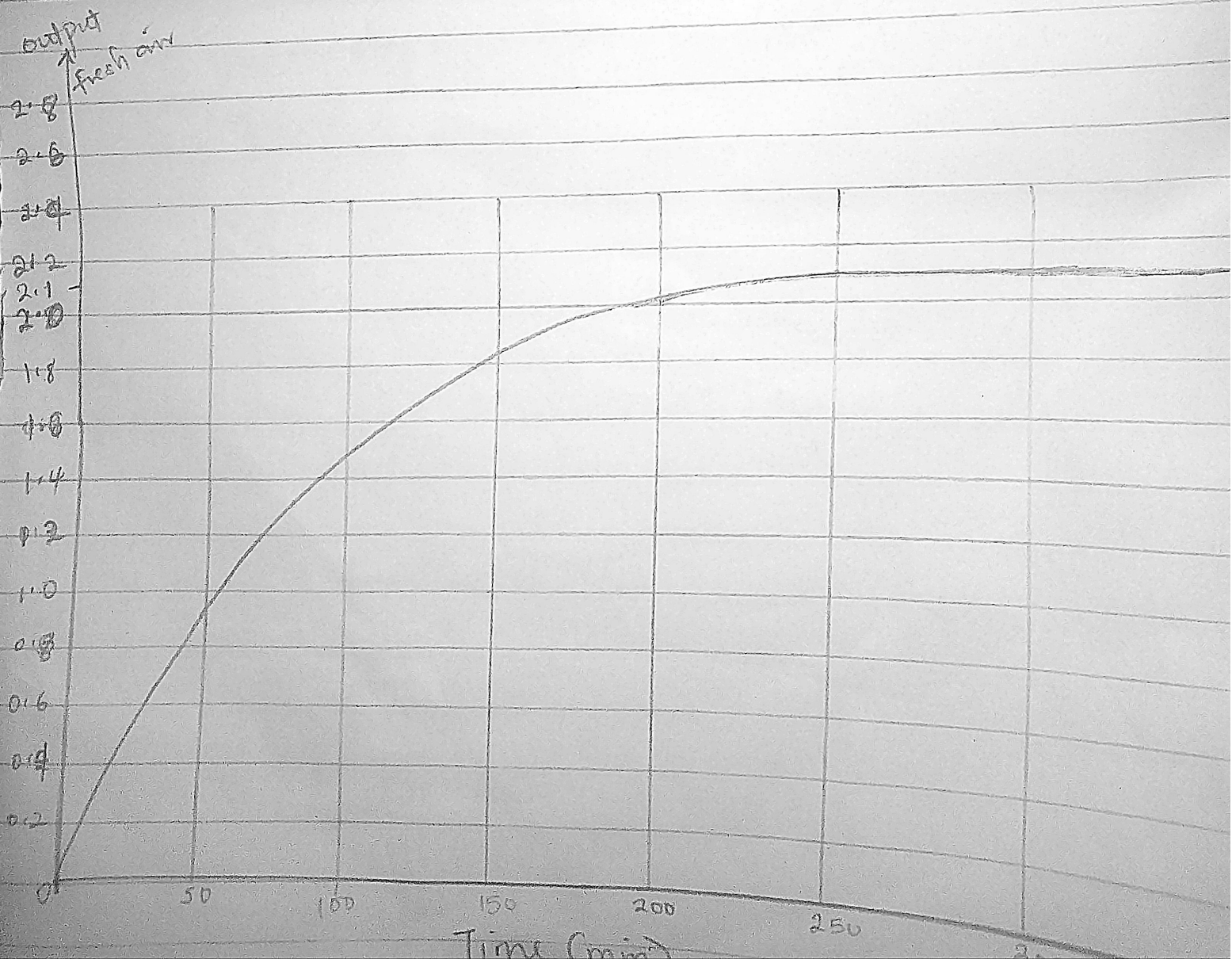
100

150

200

250

Time (min)



(d) ~~So~~ The steady state value is 20000ft^3 at 215min of the exponential approach. approach.

(e) The function shows an exponential growth to the limit of 20000ft^3 as y increases with time. The steady state value is 20000ft^3 at 215 which means after the 6 hours there was no fresh air entering or leaving the system.