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ENG 282.

* It is discovered that $600 ft^3/min$ of fresh air flows into a room containing $20,000 ft^3$ before of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of $600ft^3/min$. If the room contains no fresh air initially (a) develop a model for the amount of fresh air in the room at any time, t.

Solution:-

Let y(t) be the amount of ~~water~~ air at any time (t) in ft^3 in the room

$$\frac{dy}{dt} \rightarrow \underbrace{\text{fresh air inflow rate} - \text{fresh air outflow rate}}_{\text{Change in flow}}.$$

fresh air inflow $\rightarrow 600 ft^3/min$.

fresh air outflow $\rightarrow$ Note: The amount flowing out of the room is a function of the amount in the room.

$$\therefore \frac{600}{20000} = 0.03 min$$

i.e 0.03 of y(t) is the outflow $= 0.03y ft^3/min$

$$\therefore \frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be seperated and integrated;

$$\frac{dy}{(y-20000)} = -0.03dt$$

find the integral of both sides.

$$\ln(y - 20000) = -0.03t + c$$

$$y - 20000 = e^{(-0.03t+c)}$$

$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

Recall $C = e^c$ = ~~Integral~~ Initial equation.

$\therefore y - 20000 = e^{-0.03t} \cdot c$ —— (i).

At $t=0$, $y(t) = 0$ since the room contain no fresh air Initially

put $y=0$; $t=0$ into eqn (i).

$$y - 20000 = e^{-0.03t} \cdot c$$

$$0 - 20000 = e^{-0.03(0)} \cdot c$$

$$-20000 = e^{0} \cdot c \qquad \therefore e^0 = 1$$

$$c = -20000 \text{ —— eqn (2)}$$

put eqn (2) into (1)

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \text{ —— (3)}$$

Equation (3) above is the model for the amount of fresh air in the room.

(b) Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9.$$

$$y = 0.9 \times 20000 \quad \text{i.e } 90\% \text{ of air in the room.}$$

$$= 18000 ft^3.$$

$$y = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t}).$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9.$$

$$-0.03t = \ln(0.1).$$

$$t = \frac{\ln(0.1)}{-0.03}.$$

$$t = 77 \text{ mins}.$$

(c) With the aid of matlab, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t = 6$ hours using a step of 5 min.

NOTE: $t = 6$ hrs

$= 6 \times 60$

$= 360$ mins.

Solution:

* Command window:

* clear all
* clc
* close all
* syms y, t
* y = 20000 * (1 - exp(-0.03*t))
* t = 0:5:360
* yn = subs(y)
* plot (t, yn)
* X label ('Time (min)')
* Y label ('flow rate of fresh air (ft^3/min)')
* Grid on
* Grid minor
* Axis tight

Output. (Matlab Display).

(d) Determine the steady-state value of the amount of fresh air in the room.

Solution:

The steady-state value is $20000 ft^3$/at 215 mins (3hrs and 35mins) at exponential approach.

(e) Comment:

The functions above shows an exponential equation to the limit of $20000 ft^3$ as y increases with time. Also, when the steady-state value approaches $20000 ft^3$ at 215 mins and continues till 300min (6 hrs). The Model discussed becomes more real in gas (pneumatic technology), although tough because mixing may be imperfect.

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