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It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows into a room containing 2000 ft^3 of air. The mixture which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially,

- develop a model for the amount of fresh air in the room at any time t .
- Calculate the time at which 90% of the air in the room at any time will become fresh.
- Write the code of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6\text{hr}$ using a step time of 5min .
- Determine the steady-state value of the amount of fresh air in the room and
- Comment on the result obtained in (d)

Solution

Let $y(t)$ be the amount of air time t in (ft^3) in the room.

Fresh air inflow $\rightarrow 600 \text{ ft}^3/\text{min}$

Fresh air outflow \rightarrow Remember the amount flowing out of the room is a fraction of the amount in the room.

$$\frac{600}{20000} = 0.03 \text{ min}^{-1}$$

$\therefore 0.03 \text{ min}^{-1}$ is the fresh air outflow.

$$\begin{aligned} \frac{dy}{dt} &= 600 - 0.03y \\ &= -0.03y + 600 \\ &= -0.03(y - 20000) \end{aligned}$$

\therefore The equation can therefore be solved:

$$\frac{dy}{y - 20000} = 0.03 dt$$

Integrating both sides:

$$\ln(y - 20000) = 0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

$$y - 20000 = e^{-0.03t} \cdot e^C$$

Recall that $C = e^0$

$$y - 20000 = C e^{-0.03t}$$

$$0 = 20000 - C$$

$$C = 20000$$

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t})$$

∴ The equation above is the model for the amount of fresh air in the room.

$$(b) \quad 90\% = \frac{99}{100} = 0.9$$

$$y = 0.9 \text{ of } 20000$$

$$0.9 \times 20000$$

$$= 18000 \text{ ft}^3$$

$$y = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = 77 \text{ mins.}$$

The air in the room will be 90% fresh at 77 minutes

(c) Command window

clear all

clc

close all

Syms y, k, t

$$y = 20000 * (1 - \exp(-0.03 * t))$$

$$t = 0:5:360$$

Yn = subs(y)

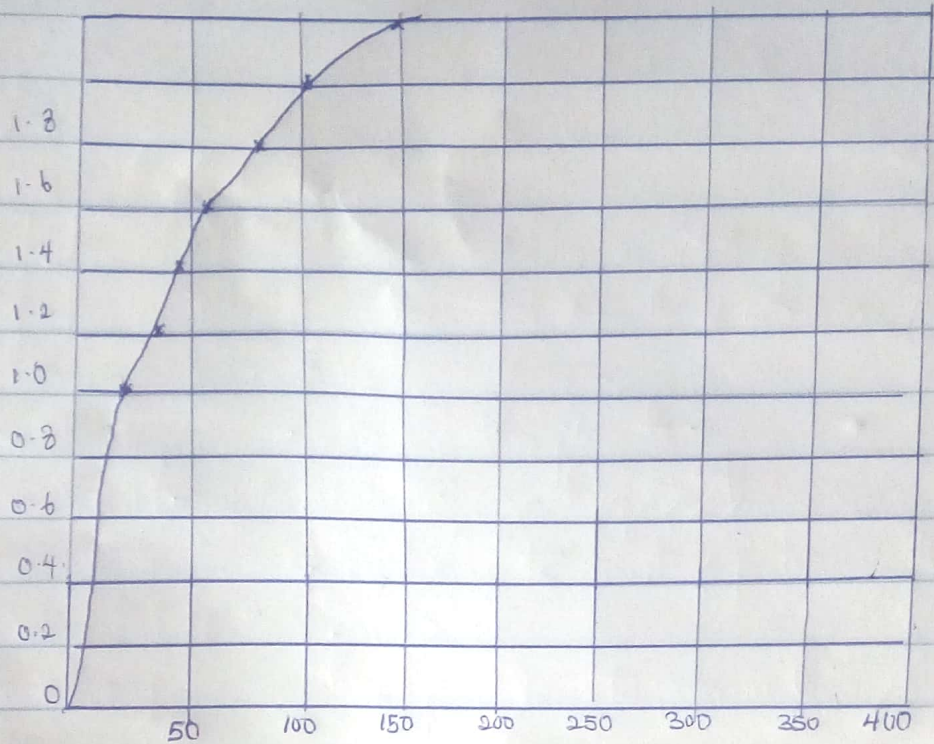
Plot(t, Yn)

xlabel('Time(mins)')

ylabel('flowrate of fresh air (ft³/min)')

grid on

grid minor



(d) The steady value is $2000 \text{ ft}^3/\text{min}$ at 215 mins of exponential approach.

(e) It shows that the limit of 2000 ft^3 of y increases with time. Also when the steady state value approaches 2000 ft^3 at 215 mins and continues till 360 minutes (1hr). The model discusses becomes more realistic in pneumatic technology although maybe because mixing may be imperfect.