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Assignment IV

It is discovered that 600 ft³/min of fresh air flows into a room containing 20000 ft³ of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially,

a) develop a model for the amount of fresh air in the room at any time t

Answer:

Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{dy}{dt} \rightarrow$ fresh air inflow rate — fresh air outflow rate

Fresh air inflow ⟶ $600 ft^3/min$

Fresh air outflow ⟶ Note: The amount flowing out of the room is a function of the amount in room

$\therefore \frac{600}{20000} = 0.03 min$

i.e 0.03 of $y(t)$ is the outflow $= 0.03y \ ft^3/min$

Now;

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

This equation can be separated and integrated

$\frac{d}{(y-20000)} = -0.03dt$

Find the integral of both sides

$\ln(y-20000) = -0.03t + c$

$y - 20000 = e^{(-0.03t + c)}$

$y - 20000 = e^{-0.03t} \cdot e^{c}$

Recall $C = e^{c}$ = initial equation

$\therefore y - 20000 = e^{-0.03t} \cdot C$ .... (1)

At $t=0$, $y(t)=0$ Since the room Contained no fresh air initially,

Put $y=0$; $t=0$ in eqn (1)

$$y = 20000 - e^{-(0.03t)} \cdot C$$

$$0 - 20000 = e^{0} \cdot C$$

$$0 - 20000 = 1(C)$$

$$C = -20000 \quad \text{-------- (2)}$$

Put eqn (2) in eqn (1)

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{.... (3)}$$

Equation (3) above is the model for the amount of fresh air in the room

b. Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100} = 0.9$$

$y = 0.9 \times 20,000$: ie 90% of air in the room

$$= 18000 ft^3$$

$$y = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{mins} \simeq 77 \text{mins}$$

c. With the aid of MATLAB, Plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6hr$ using a step time of 5min

Note. $t = 6hrs$

$$= 6 \times 60$$

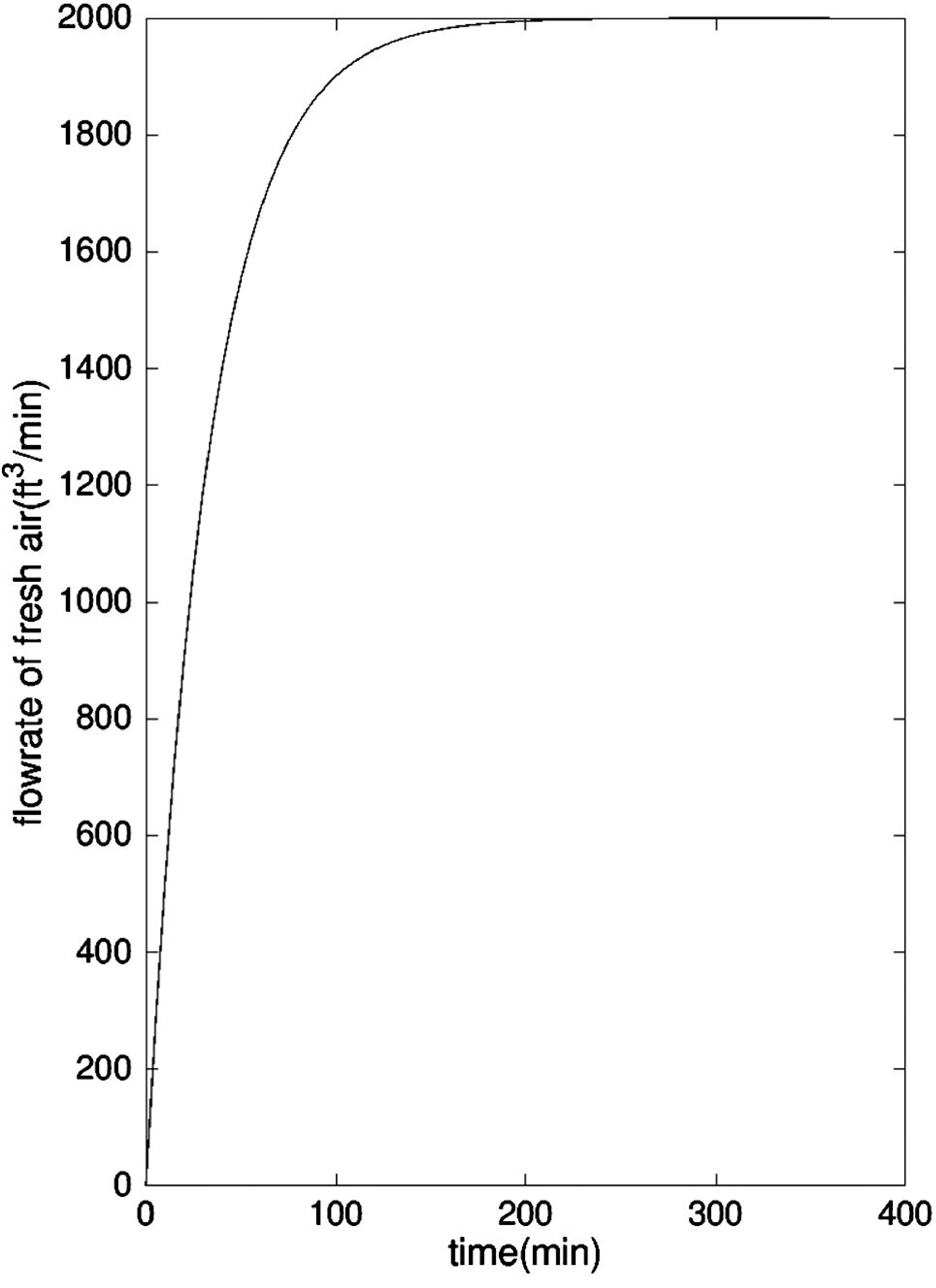
$$= 360 \text{mins}$$

Solution

Command Window

```
Clear all
clc
Close all
Syms y,t
y= 20000 * (1-exp(0.03*t))
t= 0:5:360
yn = Subs(y)
Plot (t, yn)
X Label ('Time (min)')
Y Label ('Flowrate of fresh air (ft^3/min)')
Grid On
Grid minor
Axis tight
```

Output


flowrate of fresh air(ft3/min)
time(min)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
100
200
300
400

d. determine the Steady-State Value of the amount of fresh air in the room, ~~and~~

A: The Steady-State Value is 20000 ft^3 at 215mins (3hr and 35mins) ~~of~~ Exponential ~~app~~ approach.

e. Comment on the result Obtained in (d)

The functions above Shown an Exponential Approach to the limit of 20000 ft^3 as y increases with time. Also, when the Steady State Value approaches 20000 ft^3 at 215 minutes and Continues Till 300mins (6hrs). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be Imperfect.