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17 LEAIEC31019
Civil engineering

Assignment IV

It is discovered that 600ft³/min of fresh air flows into a room containing 20000ft³ of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of 600ft³/min. If the room contains no fresh air initially (a) develop a model for the amount of fresh air in the room at any time t

Answer

Let y(t) be the amount of air at any time t in ft³ in the room

$\frac{dy}{dt}$ → Fresh air Inflow rate − Fresh air outflow rate

Fresh air inflow → 600ft³/min

Fresh air out flow → Note: The amount flowing out of the room

$\therefore \frac{600}{20000} = 0.03 min$

i.e 0.03 of y(t) is the outflow = 0.03y ft³/min

Now

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

This equation can be separated and integrated.

$\frac{dy}{(y-20000)} = -0.03dt$

Find the integral of both sides

$\ln(y - 20000) = -0.03t + c$

$y - 20000 = t(^{-0.03t+c})$

$y - 20000 = e(^{-0.03t+c})$

$y - 20000 = e^{-0.03t} \cdot e^{c}$

Recall $C = e^{c}$ = initial equation

$\therefore y - 20000 = e^{-0.03t} \cdot C$ . . . . . (1)

At t = 0, y(t) = 0 since the room contained no fresh air initially.

But $y = 0$; $t = 0$ in eqn 1

$y = 20000 = e^{-(0.03t)} C$

$0 = 20000 = e^{0} \cdot C$

$0 = 20000 = 1(C)$

$C = -20000$ ---------- (2)

Put eqn (2) in eqn (1)

$y = 20000 - 20000\, e^{-0.03t}$

$y = 20000\,(1 - e^{-0.03t})$ ... (3)

Equation (3) above is the model for the amount of fresh air in the room

b) Calculate the time at which 90% of the air in the room will become fresh,

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20{,}000$; i.e 90% of air in the room

$= 18000\ Ft^3$

$y = 20000\,(1 - e^{-0.03t})$

$0.9 = 1 \cdot e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{0.03}$

$= \frac{-2.303}{-0.03}$

$= 76.77\ mins \approx 77\ mins$

c) With the aid of matlab, plot the dynamic response of the amount of fresh air the room for $t = 0$ to $t = 6hrs$ using a step of 5 min

Note $t = 6hrs$

$6 \times 60s$

$360 mins$

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Solution

Command window

Clear all

```
cl
close all
syms y t
y = 2000*(1-exp(-0.03*t))
t = 0:5:360
yn = subs(y)
plot(t,yn)
Xlabel('Time (min)')
Ylabel('flow rate of Freon-12 (ft^3/min)')
```

Enter on

Enter number

Answer right

Output

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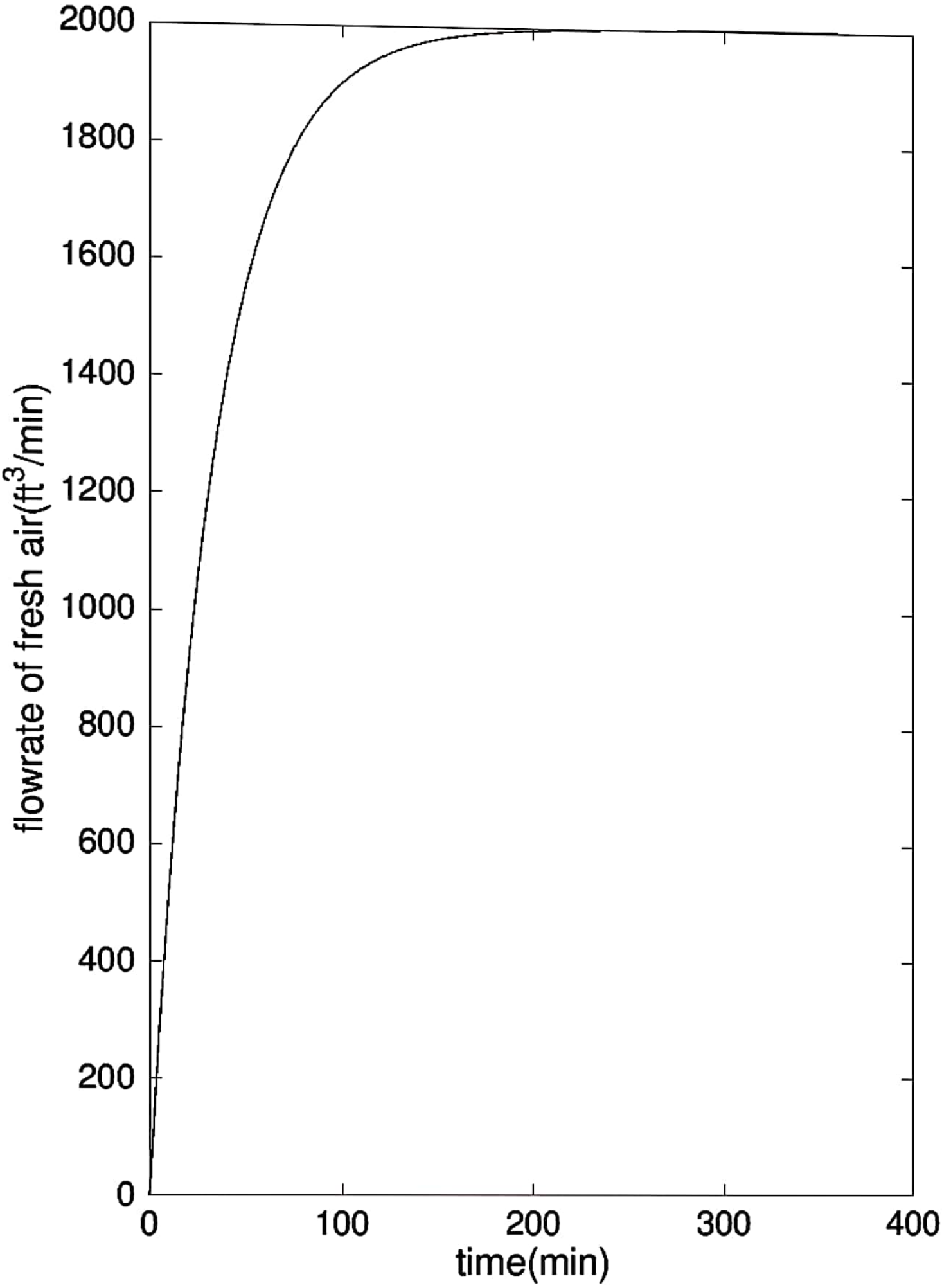

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