

ENGINEERING MATHEMATICS II

Assignment 4

Question 1

① Inflow rate = $600 \text{ ft}^3/\text{min}$

Amount of air in the room
= 20000 ft^3

Outflow rate = $600 \text{ ft}^3/\text{min} \times$

$$y(t) = \text{Inflow rate} - \text{outflow rate}$$

outflow rate = $\frac{600}{20000} \times y$

$$\text{outflow rate} = \frac{600}{20000} \times y$$

$$\text{outflow rate} = 0.03y$$

$$y(t) = \text{Inflow rate} - \text{outflow rate}$$

$$y(t) = 600 - 0.03y$$

$$y(t) = \frac{600}{0.03} (20000 - 1)$$

or change of variables

$$\frac{dy}{dt} = y(0.03(20000 - 1))$$

$$\frac{dy}{dt} = -y(0.03(1 - \frac{20000}{y}))$$

$$\frac{dy}{-y(0.03)} =$$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\int \frac{dy}{(y - 20000)} = \int -0.03 dt$$

$$\ln(y - 20000) = -0.03t + c$$

$$y - 20000 = e^{-0.03t + c}$$

② Initially $y(t) = 0$

amount of air in the room

initially = time = 0

$$y(0) = y_0 e^{-0.03t} + 20000$$

$$y(t) = y_0 e^{-0.03t} + 20000$$

$$y(0) = y_0 + 20000$$

$$y_0 = -20000$$

$$y(t) = 20000 - 20000 e^{-0.03t}$$

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$$18000 - 20000 = -20000 e^{-0.03t}$$

$$-2000 = -20000 e^{-0.03t}$$

$$0.1 = e^{-0.03t}$$

$$\ln 0.1 = -0.03t$$

$$+2.3025 = \frac{-0.03t}{-0.03}$$

$$t = \frac{76.75 \text{ min}}{0.03}$$

$$t = 76.75 \text{ min}$$

Time at which 90% of air in the room

$$= 76.75 \text{ min}$$



Command window

clear

clc

syms t

syms y

$$y = 20000 - 20000 * \exp(-0.03 * t)$$

$$t = 0:5:360$$

$$Yn = \text{subs}(y)$$

$$Ynn = \text{double}(Yn)$$

Plot (t, Ynn, 'orange')

② The value of steady state is only attained when there is no step difference in input. The ^{difference in} rate of input to output is zero. And steady state error is equals 0.

Prove

$$y(t) = 20000 - 20000e^{-0.03t}$$

$$y(t) = 20,000 - 20000e^{-0.03(180)}$$

19909 ft³ km of air that is fresh.