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Civil Engineering
17/ENG03/010

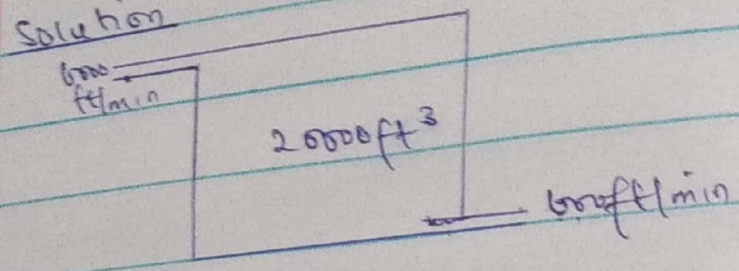
ENG 282

Assignment iv

It was discovered that 600 ft^3/min of fresh air flows in a room containing 20000 ft^3 of air. The mixture, which is made pratically uniform by circulating fans, is exhausted at a rate of 600 ft^3/min. If the room contains no fresh air initially,

a. Develop a model for the amount of fresh air in the room at any time t;
b. Calculate the time at which 90% of air in the room will become fresh.
c. With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hrs using a step time of 5 min.
d. determine the steady state value of the amount of fresh air in the room and
e. Comment on the result obtained in (d).

Solution



$\frac{dy}{dt} = y_{in} - y_{out}$

$\frac{dy}{dt}$ = Air in flow rate - Air out flow rate

$\frac{dy}{dt} = 600 ft^3/min - \frac{600}{20000} \times y$

$\frac{dy}{dt} = 600 - 0.03y$

$= 0.03(-20000 + y)$

$\frac{dy}{dt} = 0.03(y - 20000)$

$\frac{dy}{y - 20000} = (-0.03)\,dt$

Integrating both sides

$\int \frac{1}{y - 20000}\,dy = \int (-0.03)\,dt$

$\ln(y - 20000) = -0.03t + c$

Dividing thru by ln

$(y - 20000) = e^{-0.03t + c}$

$y - 20000 = e^{-0.03t} \cdot e^{c}$

$y - 20000 = e^{-0.03t} \cdot y_0$

$y - 20000 = y_0 \cdot e^{-0.03t}$

At $t = 0$, $y = 0$

$0 - 20000 = y_0 \cdot e^{-0.03(t)}$

$20000 = y_0 \cdot 1$

$y_0 = -20000 \text{ ft}^3/\text{min}$

$\therefore y - 20000 = 20000 \cdot e^{-0.03t}$

$y = 20000 - (20000 \cdot e^{-0.03t})$

b. 90% of 20000cm^3 of fresh air

$= \frac{90}{100} \times 20000$

$= 18000 \text{cm}^3$ of fresh air

When $y = 18000$, $t = ?$

$18000 = 20000 - (20000 \cdot e^{-0.03t})$

$18000 - 20000 = -(20000 \cdot e^{-0.03t})$

$-2000 = (20000 \cdot e^{-0.03t})$

$\frac{-2000}{-20000} = e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$-2.3 = -0.03t$

$t = \frac{-2.3}{-0.03}$

$t = 76.6$ mins

c. - Commandwindow

- clear
- clc
- t = 0:5:360
- y = 20000 - (20000 . e-0.03t)
- plot (t, y)
- Grid on
- Grid minor
- x label ('time')
- y label ('Volume')

* Note 60 secs → 1 min

d. The steady value is 2×10^4 ft^3 at 180 mins

e. At time t = 180 mins, the room is filled with 20000 ft^3 of fresh air and it is maintained till the 360th min [6th hour]