

Assignment 4

In going air = 600 ft³/min

let y = amount of air

~~Initial amount of air~~

Total air in room = 20000 ft³

$$\text{Outgoing air} = \frac{1}{20000 \text{ ft}^3} \times \frac{600 \text{ ft}^3}{\text{min}} y = \frac{0.03}{\text{min}} = 0.03y_0$$

$$\frac{dy}{dt} = y_{in} - y_{out}$$

$$\frac{dy}{dt} = y_{in} - \left(\frac{600}{20000} y_0 \right) = y_{in} - 0.03y_0$$

$$\frac{dy}{dt} = 600 - 0.03y_0 = -0.03(-20000 + y_0)$$

$$\frac{dy}{dt} = -0.03(y_0 - 20000)$$

$$\frac{dy}{y_0 - 20000} = -0.03 dt$$

$$\int \frac{dy}{y_0 - 20000} = -0.03 \int dt$$

$$\ln(y_0 - 20000) = -0.03t + C$$

$$y_0 - 20000 = e^{-0.03t} + e^C \quad \text{let } e^C = A$$

$$y_0 - 20000 = e^{-0.03t} + A$$

$$y_0 - 20000 = Ae^{-0.03t}$$

$$y_0 = Ae^{-0.03t} + 20000$$

at $t = 0$

$$0 = Ae^{-0.03(0)} + 20000$$

$$-20000 = A$$

$$\therefore y_0 = -20000e^{-0.03t} + 20000$$

$$\text{at } 90\% = \frac{90}{100} \times 20000 = 18000$$

$$\therefore y_c = 18000$$

$$18000 = -20000e^{-0.03t} + 20000$$

$$18000 - 20000 = -20000e^{-0.03t}$$

$$-2000 = -20000e^{-0.03t}$$

$$e^{-0.03t} = \frac{-2000}{-20000} = \frac{1}{10} = 0.1$$

$$-0.03t = \ln 0.1$$

$$-0.03t = -2.3$$

$$t = \frac{-2.3}{-0.03} = 76.75 \text{ min}$$

Using MATLAB

Command Window

clc

clear

close all

syms t y

t = 0:5:360

y = 20000 - 20000 * exp(-0.03 * t)

plot(t, y)