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Chemical Engineering
18/Engo1/027

It is discovered that $600 ft^3/min$ of fresh air flows into a room containing $20000 ft^3$ of air. The mixture which is made practically uniform by circulating fans is exhausted at a rate of 600 cubic feet per minute

If the room contain no fresh air initially

a) Develop a model for the amount of fresh air in the room
b) Calc the time at which 90% of the air in the room will become fresh
c) with matlab plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hr using a step of 5 min find the steady state value of the amount of fresh air in the room and comment on the result obtained

Sol

let $y$ = amount of air at time $t$ in $ft^3$

$\frac{dy}{dt}$ = Air inflow rate − fresh air outflow rate

fresh air inflow −− $100 ft^3/min$

air outflow −− $\frac{fy}{2000} = 0.03 y/min$

The amount flowing out of the room is a function of the amount in the room $= 0.03 y ft^3/min$

$$\frac{dy}{dt} = 600 - 0.03y$$
$$= -0.03y + 600$$
$$= -0.03(y - 20000)$$

eqn can be resolved as

$$\frac{dy}{dt} = -0.03(y - 20000)$$
$$\frac{dy}{y - 20000} = -0.03 dt$$
$$\ln(y - 20000) = -0.03t + c$$
$$y - 20000 = e^{-0.03t + c}$$
$$y - 20000 = e^{-0.03t} \cdot e^c$$
$$y - 20000 = e^{-0.03t} \cdot C \qquad (C = e^c)$$

At $t = 0$, $y(t) = 0$ since no fresh air was contained in the room

$$y - 20000 = e^{-0.03(0)} \cdot C$$
$$C = -20000$$

By substituting $C = -20000$

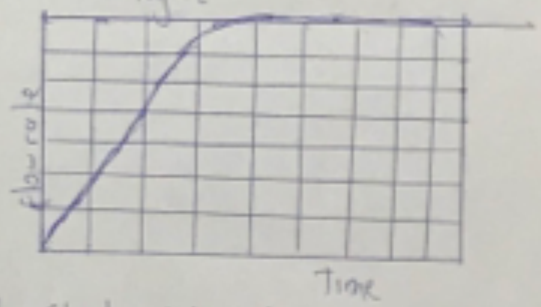
$$y - 20000 = e^{-0.03t} \cdot -20000$$
$$y = (e^{-0.03t} \cdot -20000) + 20000$$
$$y = 20000 - 20000 e^{-0.03t}$$
$$(y = 20000(1 - e^{-0.03t}))$$

↑ model for amount of fresh air

b) $90\% = \frac{90}{100} = 0.9$ $\quad y = 0.9$ of $20000 = 18000$

$$18000 = 20000(1 - e^{-0.03t})$$
$$0.9 = 1 - e^{-0.03t}$$
$$e^{-0.03t} = 1 - 0.9$$
$$-0.03t = \ln 0.1$$
$$t = \frac{-2.3026}{-0.03}$$
$$= 77 mins$$

c) With matlab
command window
clear
clc
clear all
syms t
y = 20000*(1-exp(-.03*t))
t = 0:5:360
$y_n$ = subs(y)
plot(t, $y_n$)
xlabel('time(min)')
ylabel('flow rate of fresh air')
Grid on
Grid minor
axis tight




d) The steady state value of the amount of fresh air is $20000 ft^3$ at 2.5min the exponential approach

e) The function shows an exponential growth to the limit of $20000 ft^3$ as $y$ increase with time and the steady value was $20000 ft^3$ at 2.5min it worked for 6 hours