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1/04/19 FIDE-AKWUOBI ANTHONY CHIZALU 17/ENG06/037 MECHANICAL ENGINEERING 200LVL ENG 282

It is discovered that $600 ft^3/min$ of fresh air flows into a room containing $20000 ft^3$ of air. The mixture which is made practically uniform by circulating fans is exhausted at the rate of $600 ft^3/min$. If the room contains no fresh air initially. (a) develop a model for the amount of fresh air in the room at any time (t).

Solution

(a) Let $y(t)$ be the amount of air at time (t) in the room.

Change in fresh air = Fresh air inflow rate − Fresh air outflow rate

$\therefore y' =$ Fresh air inflow rate − Fresh air outflow rate

Fresh air inflow $= 600 ft^3/min$

Fresh air outflow is a function of the amount in the room

$$\therefore \frac{600}{20000} = 0.03 \text{ of } y(t) = 0.03y \; ft^3/min$$

$$\therefore \frac{dy}{dt} = 600 - 0.03y$$

$$\frac{dy}{dt} = -0.03y + 600$$

$$\therefore \frac{dy}{dt} = -0.03(y - 20000)$$

$$\therefore \frac{dy}{(y-20000)} = -0.03dt$$

Integrating both sides

$$\therefore \int \frac{dy}{(y-20000)} = -0.03\int dt \quad \therefore \ln(y-20000) = -0.03t + c$$

$$\therefore y - 20000 = e^{(-0.03t+c)}$$

$$\therefore y - 20000 = e^{-0.03t} \cdot e^{c} \quad (\text{where } e^{c} = c)$$

$$y - 20000 = ce^{-0.03t} \quad [\text{General solution}]$$

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially

$$\therefore 0 - 20000 = ce^{-0.03(0)}$$

$$-20000 = c \cdot e^{0}$$

$$\therefore -20000 = c$$

$$\therefore c = -20000$$

$$\therefore y = 20000 - 20000e^{-0.03t} \quad [\text{Particular Solution}]$$

This is the model for the amount of fresh air in the room

$$y = 20000(1 - e^{-0.03t})$$

(b) Calculate the time at which 90% of the air in the room will become fresh.

$\therefore 90\% = \frac{90}{100} = 0.9$

$\therefore y = 0.9 \times 20000 = 18000 ft^3$

Recall $y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$\therefore 0.9 = 1 - e^{-0.03t}$

$-0.1 = -e^{-0.03t}$

$\therefore \ln(0.1) = -0.03t$

$\therefore t = \frac{-2.303}{-0.03}$

$\therefore t = 76.77 mins \simeq 77 mins.$

(c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for t = 0 to t = 6 hours using a step of 5mins.

$\therefore$ 6 hours = 6 × 60 = 360mins

Solution

```
commandwindow
clear all
clc
close all
syms y t
y = 20000 * (1 - exp(-0.03*t))
t = 0:5:360
yn = subs(y)
plot(t, yn)
xlabel('time(min)')
ylabel('flow rate of fresh air (ft^3/min)')
grid on
grid minor
axis tight
```

(d) Determine the steady...

The steady state v...

(e) Comment on result i...

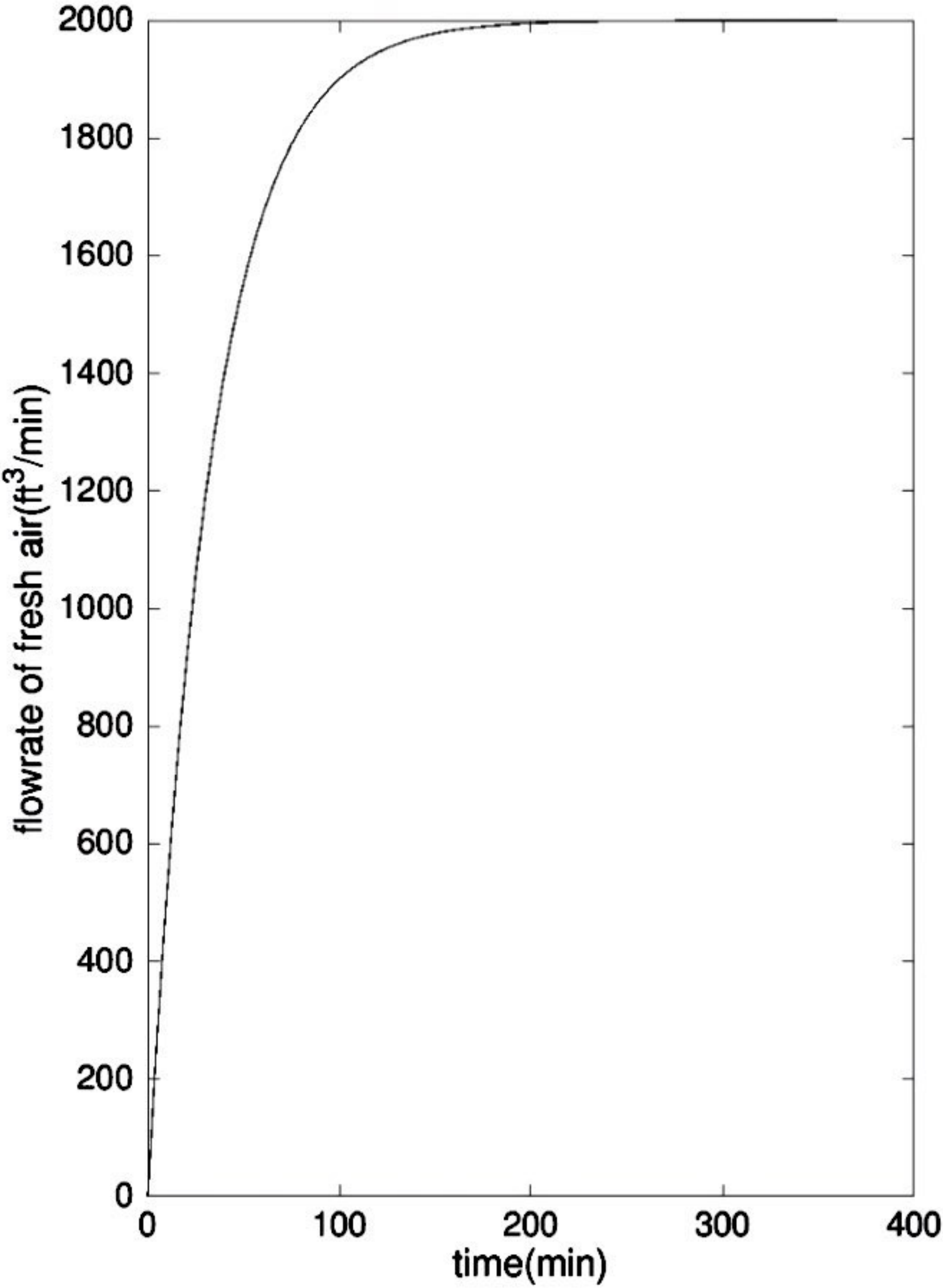
The function ...
increases with tim...
continues till 36...
maybe difficult ...

(d) Determine the steady-state value of the amount of fresh air in the room.

The steady state value is 20000ft³ at 215mins of exponential approach.

(e) Comment on result in (d)

The function shows an exponential approach to the limit of 20000ft³ as y increases with time. When the steady state value approaches 20000ft³ at 215 mins and continues till 360mins. The model becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.



flowrate of fresh air(ft^3/min)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
0
100
200
300
400
time(min)