It is discovered that $600ft^3/min$ of fresh air flows into a room containing $20000ft^3$ of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of $600ft^3/min$. The room contains no fresh air initially (a) develop a model for the amount of fresh air in the room at any time, t.

Solution

Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{\partial y}{\partial t} \rightarrow$ fresh air inflow rate − fresh air outflow rate

Fresh air inflow $\longrightarrow 600ft^3/min$

fresh air outflow $\rightarrow$ Note: The amount flowing out of the room is a function of the amount in the room.

$$\therefore \frac{600}{20000} = 0.03min$$

i.e 0.03 of $y(t)$ is the outflow $= 0.03yft^3/min$

Now;

$$\frac{\partial y}{\partial t} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 2000)$$

This equation can be separated and integrated;

$$\frac{\partial y}{(y-20000)} = -0.03\partial t$$

Find the integral of both sides

$$\ln(y-20000) = -0.03t + c$$

$$y - 20000 = e^{(-0.03t + c)}$$

$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

Recall $C = e^{c}$ = initial equation

$\therefore y - 20000 = e^{-0.03t}\, c$ ——— (1)

At $t = 0$, $y(0) = 0$ since the room contained no fresh air initi...

Put $y = 0$; $t = 0$ in eqn 1

$y - 20000 = e^{-(0.03 \times 0)} \cdot c$

$0 - 20000 = e^{0} \cdot c$

$0 - 20000 = 1(c)$

$c = -20000$ ——— (2)

Put eqn (2) in eqn (1)

$y = 20000 - 20000\, e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ——— (3)

Equation (3) above is the model for ~~the~~ amount of fresh air in the room

b Calculate the time at which 90% of the air in the room will become fresh

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000$; i.e 90% of air in the room

$= 18000\, ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = \underline{77 \text{ mins}}$

c With the aid of matlab, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hrs with a step of 5min

Note: $t = 6$ hrs

$= 6 \times 60$

$= 360$ mins

Solution

Command window

```
Clear all
clc
close all
Syms y,t
y= 20000*(1- exp(-0.03*t))
t= 0:5:360
yn = subs(y)
Plot (t,yn)
X label ('Time(min)')
Y label ('Flowrate of fresh air (ft^3/min)')
Grid on
Grid minor
Axis tight
```

Output

d Determine the steady-state value of the amount of fresh air in the room.

Answer: The steady-state value is 20000ft³/at 215 mins (3hrs and 35mins) at exponential approach.

e Comment: The functions above shows an exponential equation to the limit of 20000ft³ as y increases with time. Also, when the steady-state value approaches 20000ft³ at 215 minutes and continues. Till 300mins (5hrs). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.

$\therefore y - 20000 = e^{-0.03t} \cdot c$ ——— (1)

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially

Put $y = 0$; $t = 0$ in eqn 1

$y - 20000 = e^{-0.03(0)} \cdot c$

$0 - 20000 = e^{0} \cdot c$

$0 - 20000 = 1(c)$

$c = -20000$ ——— (2)

Put eqn (2) in eqn (1)

$y = 20000 - 20000 e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ——— (3)

Equation (3) above is the model for the amount of fresh air in the room

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$18000 = 20000(1 - e^{-0.03t})$

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$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = \underline{77 mins}$

c With the aid of matlab, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6hrs$ using a step of 5min

Note; $t = 6hrs$

$= 6 \times 60$

$= 860 mins$