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17/ENG02/028

It is discovered that 600ft^3/min of fresh air flows into a room containing 20000ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute. If the room contain no fresh air initially.

a. develop a model for the amount of fresh air in the room at any time t

b. Calculate the time at which 90% of the air in the room will become fresh.

c. with the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6hr$ using a step time of 5min.

d. determine the steady state value of the amount of fresh air in the room and

e. comment on the result obtained in (d)

a. Let y be the amount of air at time t in (ft^3) in the room

$$\frac{dy}{dt} = \text{Air inflow rate} - \text{fresh air outflow rate}$$

fresh Air inflow $\longrightarrow$ 600ft^3/min..

fresh air outflow $\longrightarrow \dfrac{600}{2000} = 0.03 min$

The amount flowing out of the room is a function of the amount in the room

$= 0.03y$ ft^3/min

$$\frac{dy}{dt} = 600 - 0.03y = -0.03y + 600 = -0.03(y - 20000)$$

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Equation can be resolved as

$$\frac{dy}{dt} = -0.03(y-20000)$$

$$\frac{dy}{y-20000} = -0.03dt \qquad \ln(y-20000) = -0.03t + c$$

$$\ln(y-20000) = -0.03t + c$$
$$= y - 20000 = e^{-0.03t+c}$$
$$y - 20000 = e^{-0.03t} \cdot e^{c}$$

At $t=0$, $y(t)=0$ Since no fresh air was contained in the room initially

$$y - 20000 = (e^{-0.03(0)}$$
$$0 - 20000 = c$$
$$c = -20000$$

Substituting $c = -20000$

$$y - 20000 = e^{-0.03t} \cdot -20000$$
$$y = (e^{-0.03t} \cdot 20000) + 20000$$
$$y = 20000 - 20000e^{-0.03t}$$
$$y = 20000(1-e^{-0.03t})$$

This is the model for the amount of fresh air in the room

b. Calculate the time at when 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \text{ of } 20000$$
$$0.9 \times 20000 = 18000ft^3$$

$$\therefore 18000 = 20000(1-e^{-0.03t})$$
$$18000 = 20000(1-e^{-0.03t})$$
$$0.9 = 1-e^{-0.03t}$$
$$e^{-0.03t} = 1-0.9 = 0.1$$

$-0.03t = \ln(0.1)$

$t = \frac{-2.3026}{-0.03}$

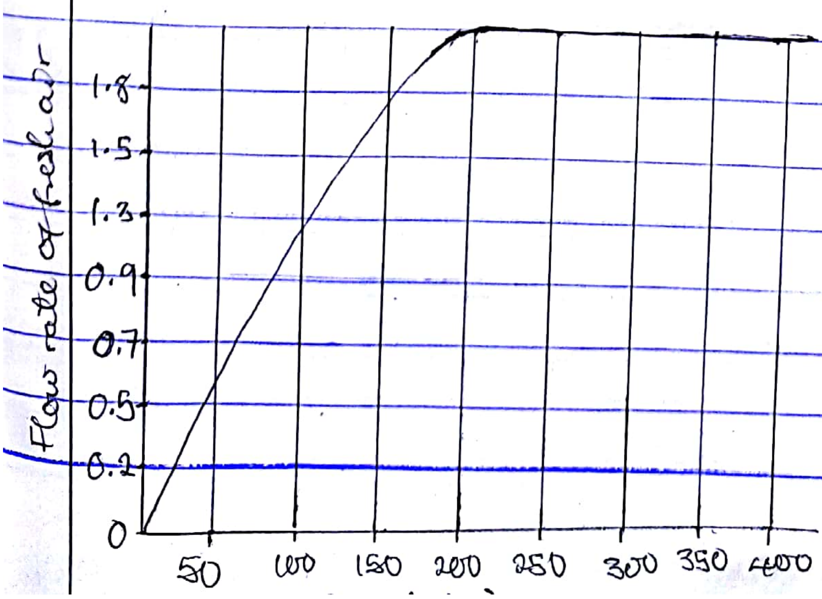
$t = 77 min$

c. Plotting the dynamic response of amount of fresh air with the aid of MATLAB

CODE

```
commandwindow
clear
clc
close all
symst
y = 20000*(1-exp(-0.03*t))
t=0:5:360
yn = Subs(y)
plot(t,yn)
xlabel('Time(min)')
ylabel('flowrate of fresh air')
grid on
grid minor
axis tight
```

Output



d. The steady state-value is $20000 ft^3$ of 215min of the exponential approach

e. The steady state-value proves the function which shows the exponential growth of limit of $20000 ft^3$ as y increase with time.