

Maths assignment 1/1/18 Solutions

@ let $y(t)$ be the amount of air time t in the room

$$\frac{dy}{dt} = \text{air inflow rate} - \text{fresh air outflow rate}$$

fresh air inflow = $600 \text{ ft}^3/\text{min}$.

fresh air outflow = $\frac{600}{20000} = 0.03 \text{ min}^{-1}$.

i.e. 0.03 of $y(t)$ in the outflow = $0.03y \text{ ft}^3$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

factorize

$$\frac{dy}{dt} = -0.03(y - 20,000)$$

By using separation of variables

$$\frac{dy}{y - 20,000} = -0.03 dt$$

$$y - 20,000$$

$$\int \frac{dy}{y - 20,000} = \int -0.03 dt$$

$$\ln|y - 20,000| = -0.03t + c$$

$$y - 20,000 = e^{-0.03t + c}$$

$$y - 20,000 = e^{-0.03t} + e^c$$

Recall $c = e^c$

$$y - 20,000 = e^{-0.03t} C \quad \text{--- (1)}$$

At $t=0$, $y(t)=0$

$$y - 20000 = C e^{-0.03t}$$

$$0 - 20000 = C$$

$$C = -20000 \quad \dots \textcircled{1}$$

Sub $\textcircled{1}$ into $\textcircled{1}$

$$y - 20,000 = e^{-0.03t} (-20,000)$$

$$y = -20000 e^{-0.03t} + 20,000$$

$$= 20000 [-e^{-0.03t} + 1]$$

$$= 20000 [1 - e^{-0.03t}] \quad \dots \textcircled{2}$$

Eqn above is the model for the amount of fresh air in the room

$$\textcircled{b} \quad 90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \text{ of } 20000$$

$$= 0.9 \times 20,000$$

$$= 18000 \text{ ft}^3$$

$$\text{Recall } y = 20,000 (1 - e^{-0.03t})$$

$$18000 = 20,000 (1 - e^{-0.03t})$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03} = 76.75 \approx 77 \text{ mins.}$$

$$-0.03$$

\therefore The air in the room will be 90% of fresh at 77 minutes.

① Command window

Clear all

clc

close all

Syms y, t, k

$$y = 20000 * (1 - \exp(-0.03 * t))$$

$$t = 0: 5: 360$$

yt = Subs (y)

Plot (t, yt)

x label ('Time (min)')

y label ('flow rate of fresh Air (ft³/min)')

grid on

grid minor

axis tight

d) Steady ~~flow~~ state value is 20,000 ft³/min at 215 min
at exponential approach

② The function shows an exponential approach to the limit at 20,000 ft³ of y increases with time. Also with the steady state value approach 20,000 ft³ at 215 min and continues for 360 minutes (hr). The model then discusses becoming more realistic in pneumatic technology although maybe difficult because mixing may be imperfect.