

Question 2

$$\text{Volume of the room} = 20000 \text{ ft}^3$$

$$\text{Inlet} = 600 \text{ ft}^3/\text{min}$$

$$\text{Outlet} = 600 \text{ ft}^3/\text{min}$$

$$\frac{dy}{dt} = A_{in} - A_{out}$$

$$\text{where } A_{out} = \frac{600}{20000} y$$

$$A_{in} = 600 \text{ ft}^3/\text{min}$$

$$\frac{dy}{dt} = 600 - \left(\frac{600}{20000} y \right)$$

$$\frac{dy}{dt} = (600 - 0.03y)$$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

Collecting like terms

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

$(y - 20000)$

Integrate both sides

$$\int \frac{dy}{(y - 20000)} = \int -0.03 dt$$

$$\ln(y - 20000) = -0.03t + C$$

$$y(t) - 20000 = e^{-0.03t + C} \quad \text{where } e^C = A$$

$$y(t) - 20000 = e^{-0.03t} \cdot e^C$$

$$y(t) - 20000 = A e^{-0.03t}$$

$$y(t) - 20000 = A e^{-0.03t}$$

$$y(t) = 20,000 + A e^{-0.03t}$$

at time, $t=0$, $y(0) = 0$

$$\therefore y(0) = 20,000 + A e^0$$

$$0 = 20,000 + A$$

$$A = -20,000$$

$$y(t) = 20,000 - 20,000 e^{-0.03t}$$

b) The time taken for 90%

$$\frac{90}{100} \times 20,000$$

$$y(t) = 18,000$$

$$18,000 = 20,000 - 20,000 e^{-0.03t}$$

$$-2,000 = -20,000 e^{-0.03t}$$

$$0.1 = e^{-0.03t}$$

$$-2.30 = -0.03t$$

$$t = 76.67 \text{ min}$$

c) (i) Command Window

(ii) CLC

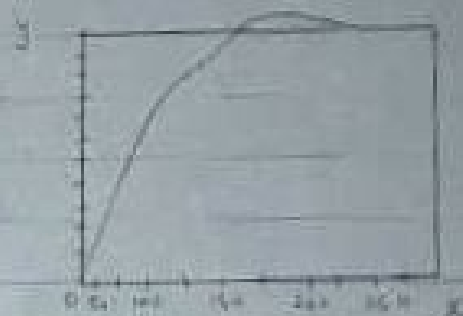
(iii) Clear

(iv) Syms y t

(v) t = 0:0.5:360

(vi) y = 20,000 - 20,000 * exp(-0.03 * t)

(vii) Plot (t, y)



d) Z at 18000

e) At the rate at which fresh air flows into the room will be completely filled with fresh air in about 3 hours.