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Civil Engineering
ENG 282

It is discovered that $600 \text{ ft}^3/\text{min}$ of fresher flows into a room containing $20,000 \text{ ft}^3$ of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of $600 \text{ ft}^3/\text{min}$. If the room contains no fresh air initially, (a) develop a model for the amount of fresh air in the room at any time, t .

Solution

(Let $y(t)$ be the amount of air at any time t in ft^3 in the $\frac{dy}{dt} \rightarrow$ fresh air inflow rate - fresh air outflow rate at

fresh air inflow $\rightarrow 600 \text{ ft}^3/\text{min}$

fresh air outflow \rightarrow rate: the amount flowing out of the room is a fraction of the amount in the room

$$\therefore \frac{600}{20000} = 0.03 \text{ min}^{-1}$$

Now,

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be separated and integrated,

$$\frac{dy}{(y-20000)} = -0.03 dt$$

find the integral of both sides

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{(-0.03t + C)}$$

$$y - 20000 = e^{-0.03t} \cdot e^C$$

Recall $C = e^C =$ initial equation

$$\therefore y - 20000 = e^{-0.03t} \cdot C \quad \text{--- (1)}$$

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially

put $y = 0$, $t = 0$ in eqn (1)

$$y - 20000 = e^{-0.03t} \cdot C$$

$$0 - 20000 = e^0 \cdot C$$

$$0 - 20000 = C$$

$$C = -20000 \quad \text{--- (2)}$$

put eqn (2) in eqn (1)

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{--- (3)}$$

Equation (3) above is the model for the amount of fresh air in the room

③ Calculate the time at which 90% of the air in the room will become

$$90\% = \frac{90}{100} = 0.9$$

$$\rightarrow 0.9 \times 20000; \text{ i.e. } 90\% \text{ of air in the room} \\ = 18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$t = \underline{77 \text{ mins}}$$

④ Write the code of matlab, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=$ this using a step of 5 min

$$\text{Note: } t = 6 \text{ hrs}$$

$$= 6 \times 60$$

$$= 360 \text{ mins.}$$

Solution

Command Window

clear all

clc

close all

Syms ~~y~~ y, t

$$y = 20000 \cdot (1 - \exp(-0.03 \cdot t))$$

$$t = 0:5:360$$

$$y_n = \text{subs}(y)$$

plot(t, y_n)

x-label ~~Time~~ ('Time (min)')

y-label ('flowrate of fresh air (ft³/min)')

Grid on

Grid minor

Axis tight

⊙

Output

Q Determine the steady-state value of the amount of fresh air in the room.

Answer: The steady-state value is $20000 \text{ ft}^3/\text{min}$ (2 hrs and 35 mins) at exponential approach

② comment: The functions above ^{shows} ~~shows~~ an exponential equation to the limit of $20,000 \text{ ft}^3$ as γ increases with time. Also when the steady state ~~continues~~ values approaches $20,000 \text{ ft}^3$ at 215 minutes and continues. At 300 minutes (6 hours) the model discussed ~~can~~ becomes more relative in pneumatic technology, although ~~monitored~~ maybe difficult because mixing may be imperfect.