

Eq 2 can be resolved as

$$\frac{dy}{dt} = -0.03(y - 20,000)$$

$$\frac{dy}{y - 20,000} = -0.03 dt \quad \therefore \ln(y - 20,000) = -0.03t + C$$

$$y - 20,000 = e^{-0.03t + C}$$

$$y - 20,000 = e^{-0.03t} \cdot e^C \quad (\text{According to rules of indices})$$

$$\text{At } t = 0, y(t) = 0$$

Since no fresh air was contained in the room initially

$$y - 20,000 = C e^{-0.03(0)}$$

$$0 - 20,000 = C$$

$$C = -20,000$$

$$\text{Sub } C = -20,000$$

$$y - 20,000 = e^{-0.03t} - 20,000$$

$$y = 20,000 (1 - e^{-0.03t})$$

This is the model for the amount of fresh air in the room

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Elect/Elect

A Let y be the amount of air at time, t in Gft^3 in the room

$\frac{dy}{dt} \Rightarrow$ Air in flow rate - fresh air out from rate

Fresh air inflow = $100 \text{ft}^3/\text{min}$

Fresh air outflow = 600

$2000 = 0.3 \text{ min}$

The amount flowing out of the room is a function of the amount in the room = $0.03y \text{ft}^3/\text{min}$

$\frac{dy}{dt} = 600 - 0.03y = -0.03y + 600 = -0.03(y - 20,000)$

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Output

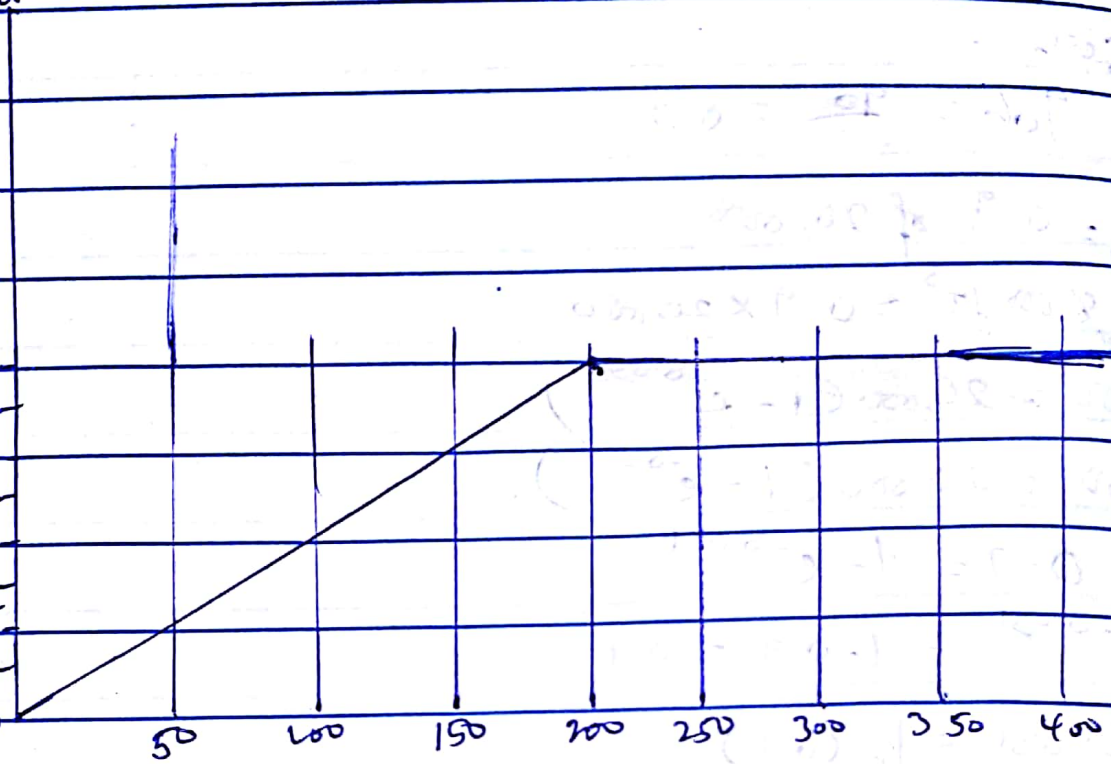
A
in

flow rate of fresh air

2.2
1.8
1.5
1.3
0.9
0.7
0.5
0.2
0

50 100 150 200 250 300 350 400

m



SA/04/035
Time, t in (ft³)

out from rate

ction of the amount

0.03(y - 20,000)

0.03t + c

s of (ndres)

initially

the room

e

B. Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \text{ of } 20,000$$

$$18,000 \text{ ft}^3 = 0.9 \times 20,000$$

$$\therefore 18,000 = 20,000 (1 - e^{-0.03t})$$

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$$\cancel{20,000} \cdot 0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9 = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = 77 \text{ min.}$$

C) Command window

clear

clc

close all

Syms t

$$y = 20,000 * [1 - \exp(-0.03 * t)]$$

$$t = 0 : 5 : 360$$

$$y_n = \text{subs}(y)$$

plot (t, y_n)

xlabel ['Time (min)']

grid on

grid minor

axis tight

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