d) Determine the steady-state value of the amount of fresh air in the room

The steady state value is 20000 ft^3 at 215 mins of exponential approach.

e) Comment on result in (d)

The function shows an exponential approach to the limit of 20000 ft^3 as y increases with time. When the steady state value app-roaches 20000 ft^3 at 215 mins and continues till 360 mins. The mo-del becomes more realistic in pneumatic technology, althoug maybe difficult because mixing may be imperfect

B) Calculate the time at which 90% of the air in the room will become fresh

$\therefore 90\% = 90/_{100} = 0.9$

$\therefore y = 0.9 \times 20000 = 18000 ft^3$

Recall $y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$\therefore 0.9 = 1 - e^{-0.03t}$

$+0.1 = +e^{-0.03t}$

$\therefore \ln(0.1) = -0.03t$

$\therefore t = \frac{-2.303}{-0.03}$

$\therefore t = 76.77 mins \simeq 77 mins.$

(C) With the aid of MATLAB, Plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hours using a step of 5mins

$\therefore 6 hours = 6 \times 60 = 360 mins$

Solution

Commandwindow

```
clear all
clc
Close all
Symsyt
y = 20000 * (1 - exp(-0.03 * t))
t = 0:5:360
yn = subs(y)
Plot(t, yn)
xlabel('time(min)')
ylabel('flow rate of fresh air (ft^3/min)')
grid on
grid minor
axis tight
```

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ELECT/ELECT

a) Let $y(t)$ be the amount of air at time (t) in the room

Change in fresh air = Fresh air inflow rate − Fresh air outflow rate

∴ y = Fresh air inflow rate − Fresh air ~~outflow~~ outflow rate

Fresh air in flow = $600 \, ft^3/min$

Fresh air outflow is a function of the amount in the room

$$\therefore \frac{600}{20000} = 0.03 \, y(t) = 0.03y \, ft^3/min$$

$$\therefore \frac{dy}{dt} = 600 - 0.03y$$

$$\frac{dy}{dt} = -0.03y + 600$$

$$\therefore \frac{dy}{dt} = -0.03(y - 20000)$$

$$\therefore \frac{dy}{(y-20000)} = -0.03dt$$

Integrating both sides

$$\therefore \int \frac{dy}{(y-20000)} = -0.03 \int dt \quad \therefore \ln(y - 20000) = -0.03t + c$$

$$\therefore y - 20000 = e^{(-0.03t + c)}$$

$$\therefore y - 20000 = e^{-0.03t} \cdot e^{c} \quad (\text{where } e^{c} = C)$$

$$y - 20000 = Ce^{-0.03t} \quad [\text{General solution}]$$

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially

$$\therefore 0 - 20000 = Ce^{-0.03(0)}$$

$$-20000 = C \cdot e^{0}$$

$$\therefore -20000 = C$$

$$\therefore C = -20000$$

$$\therefore y = 20000 - 20000e^{-0.03t} \quad [\text{Particular solution}]$$

∴ This is the model for the amount of fresh air in the room.

$$\therefore y = 20000(1 - e^{-0.03t})$$