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17/ENG02/1041
COMPUTER ENGINEERING
ENGINEERING MATHS

Maths Assignment 4

Solution
Let $y(t)$ be the amount of air at anytime t in the room dy/dx → fresh air inflow rate - fresh air outflow rate fresh air inflow → 600 ft 3/min
fresh air outflow → Note: The amount flowing out of the room is a function of the amount in the room

$\frac{600}{10000} = 0.03$ mm
i.e 0.03 of $y(t)$ is the outflow $= 0.03$ y ft^3/min

Now

$$\frac{dy}{dt} = 600 - 0.03y$$
$$= -0.03y + 600$$
$$= 0.03(y - 20000)$$

This equation can be separated and integrated

$$\frac{dy}{(y - 20000)} = -0.03\,dt$$

find the integral of both sides

$$\ln(y - 20000) = -0.03t + C$$
$$y - 20000 = e^{(-0.03t + C)}$$
$$y - 20000 = e^{-0.03t} \cdot e^{C}$$

Recall $C = e^{C}$: initial equation

$$\therefore y - 20000 = e^{-0.03t}\,C \quad ---1$$

At $t = 0$, $y(t) = 0$, since the room contained no fresh air initial Put $y = 0$; $t = 0$ in eqn 1

$$y = -20000 = e^{-0.03t}\,C$$
$$0 - 20000 = e^{0}C$$

$0 - 20000 = 1(c)$

$C = -20000 \quad --- 2$

Put eqn (2) in eqn 1

$y = 20000 - 20000 e^{-0.03t}$

$y = 20000 \cdot (1 - e^{-0.03t}) \quad --- 3$

equation (3) above is the model for the amount of fresh air in the room

B) Calculate the time at which 90% of the air in the room will become

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000$; i.e 90% of air in the room

$= 18000 ft^3$

$y = 20000 (1 - e^{-0.03t})$

$18000 = 20000 (1 - e^{-0.03t}$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = 77 mins$

c) With the aid of matlab, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6hrs$. Using and step of 5mins Note: $t = 6hrs = 6 \times 60 = 360 mins$

Solution

Command window

```
clear all
clc
Close all
syms y,t
```

```
y= 20000(1 -exp(0.03t))
   t=0:5:360
   yn=Subs(y)
   plot(t, yn)
X table(Prime(min))
X'label(flowrate of fresh air(ft^3/min
   Grid DM
Grid minor
   Axis Height
```

Output

D) Determine the Steady-State value of the amount of fresh air in the room

Solution

The Steady-State Value is 20000 ft^3 lat 215mins (3hrs and 35mins) or Exponential approach

E) Comment: The function above shows an exponential equation to the limit of 20000 ft^3, as y increase with time, Also, when the Steady-State value approaches 20000 ft^3 at 215min and continues for 360min (6hrs). The model disease becomes more realistic in Pneumatic technology, although maybe difficult because many may be imperfect