

Matric no: 17/ENG04/038

Electrical electronics engineering

ENE282 ASSIGNMENT 4

It is discovered that 600 ft³/min of fresh air flows into a room containing 20000 ft³ of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially.

- develop a model for the amount of fresh air in the room at any time, t .
- Calculate the time at which 90% of the air in the room will become fresh.
- with the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6$ hr using a step time of 6 min.
- determine the steady-state value of the amount of fresh air in the room, and
- comment on the result obtained in (d).

Solution.

Let y be the amount of air at time, t in (ft³) in the room

$$\frac{dy}{dt} = \text{Fresh air inflow rate} - \text{Fresh air outflow rate}$$

fresh air inflow $\rightarrow 100 \text{ ft}^3/\text{min}$
fresh air outflow $\rightarrow 600$
 $2000 = 0.03 \text{ min}$

The amount flowing out of the room is given a function of the amount in the room.
 $= 0.03y \text{ ft}^3/\text{min}$

$$\frac{dy}{dt} = 600 - 0.03y$$
$$= -0.03y + 600$$
$$= -0.03(y - 20000)$$

Equation can be resolved as

$$\frac{dy}{(y - 20000)} = -0.03 dt$$
$$\ln(y - 20000) = -0.03t + C$$

$$\ln(V - 20000) = -0.03t + C$$

$$= V - 20000 = e^{-0.03t + C}$$

$$= V - 20000 = e^{-0.03t} \cdot e^C$$

$C = e^C = \text{initial condition.}$

$$= V - 20000 = e^{-0.03t} \cdot C$$

At $t=0, V(t)=0$ Since no fresh air was contained in the room

initially
 $V - 20000 = C(e^{-0.03(0)})$

$$0 - 20000 = C$$

$$C = -20000$$

Re. Substituting $C = -20000$

$$V - 20000 = e^{-0.03t} \cdot -20000$$

$$V = (e^{-0.03t} \cdot -20000) + 20000$$

$$V = 20000 - 20000e^{-0.03t}$$

$$V = 20000(1 - e^{-0.03t})$$

The equation above is the model for the amount of fresh air in the room

- b. Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100} = 0.9$$

$$V = 0.9 \text{ of } 20000$$

$$0.9 \times 20000 = 18000 \text{ ft}^3$$

$$\therefore 18000 = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$e^{-0.03t} = \ln 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{-2.3026}{-0.03} \quad t = 77 \text{ mins}$$

With the aid of matlab.

Command window

Clear ~~all~~

clc

Close all

Syms t

$$y = 20000 * (1 - \exp(-0.03 * t))$$

t = 0:5:360

y_n = Subs(y)

Plot (t, y_n)

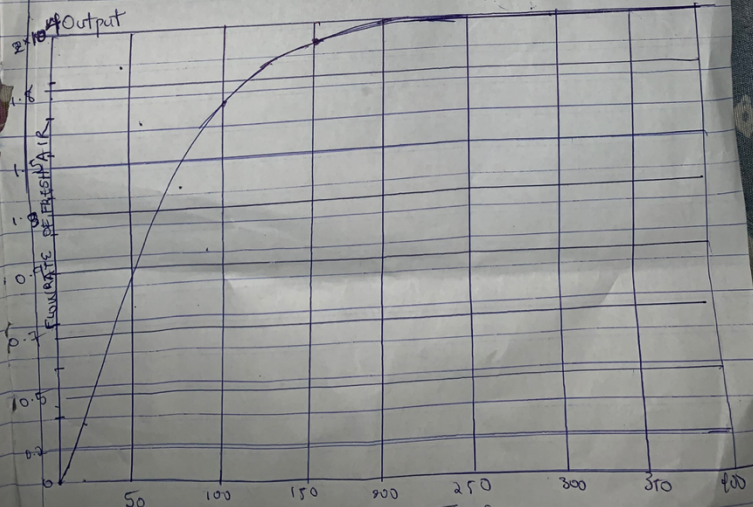
Xlabel ('Time (min)')

Ylabel ('Flowrate of fresh air')

grid on

grid minor

axis tight



d) Determine the steady-state value of the amount of fresh air

The steady state-value is 20000 ft^3 at 215 min of the exponential approach.

e) Comment on the result obtained in d)

The function shows an exponential growth / approach to the limit of 20000 ft^3 as t increases with time and the steady value was 20000 ft^3 at 215 min it worked for 6 hours.