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17/ENG06/021
MECHANICAL ENGINEERING
ENG 282

SOLUTION

$dy/dt$ = Air inflow rate - Air outflow rate

$dy/dt = y_{in} - y_{out}$

$\frac{dy}{dt} = 600 ft^3/min - \frac{600}{20000} \times y$

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03 (-20000 + y)$

$dy/dt = -0.03 (y - 20000$

$dy/y - 20000 = (-0.03) dt$

Integrating Both Sides

$\int \frac{1}{y - 20000} dy = \int (0.03) dt$

$\ln (y - 20000) = -0.03t + C$

Dividing Both Sides by $\ln$

$(y - 20000) = e^{-0.03t + C}$

$y - 20000 = e^{-0.03t} \cdot e^C$

$y - 20000 = e^{-0.03t} \cdot y_0$

$y - 20000 = y_0 e^{-0.03t}$

At time $t = 0$, $y = 0$

$0 - 20000 = y_0 \cdot e^{-0.03(0)}$

$-20000 = y_0 \cdot 1$

$y_0 = -20000 \, ft^3/min$

$\therefore y - 20000 = -20000 \cdot e^{-0.03t}$

$y = 20000 - (20000 \cdot e^{-0.03t})$

90% of $20000 cm^3$ of fresh air

$= 90/100 \times 20000$

$= 18000 cm^3$ of fresh air

When $y = 18000$, find $t$

$18000 = 20000 - (20000 \cdot e^{-0.03t})$

$18000 - 20000 = -(20000 \cdot e^{-0.03t})$

$-2000 = -(20000 \cdot e^{-0.03t})$

$\frac{-2000}{-20000} = e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$-2.3 = -0.03t$

$t = \frac{-2.3}{-0.03}$

$t = 76.6 mins$

Command Window

clear

clc

close all

- $t = 0:5:360$
- $y = 20000 - (20000 \cdot e^{-0.03t})$

Plot (t, y)

Grid on

Grid minor

xlabel('time')
ylabel('Volume')

The steady value is $2 \times 10^4$ $ft^3$ at 180 mins

At time t = 180 mins, the room is filled with 20000 $ft^3$ of fresh air and it is maintained till the 360th min (6th hr)