1 NAME: ODUNEYE LEONARD MOYOSOREOLUWA

MATRIC NO: 17/ENG02/061

DEPARTMENT: COMPUTER ENGINEERING

It is discovered that 600 ft^3/min of fresh air flows into a room containing 20000 ft^3 of air. The mixture, which is made practically uniform by circulating fans is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially

(a) develop a model for the amount of fresh air in the room at any time, t.

To develop the model for the amount of fresh air in the room at any time, t

Let y(t) be the amount of air in ft^3 at any time (t) in the room.

Therefore, $\frac{dy}{dt}$ = fresh air inflow rate - fresh air outflow rate ...(1)

From the information provided, fresh air inflow rate = 600 ft^3/min

Fresh air outflow rate:

The amount of fresh air flowing out the room is a function of the amount flowing into the room

Therefore, $\frac{600}{20000} = 0.03 \text{min}$

0.03 of y(t) is the amount of air flowing out = 0.03 y ft^3/min

Rewriting equation (1)

$$\frac{dy}{dt} = 600 - 0.03y \quad ...(2)$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000) \quad ...(3)$$

Integrating and seperating eqn (3)

$$\frac{dy}{(y-20000)} = -0.03dt$$

$$\int \frac{dy}{(y-20000)} = \int -0.03dt$$

$\ln(y-20000) = -0.03t+C$

$y-20000 = e^{(-0.03t+C)}$

$y-20000 = e^{-0.03t} \cdot e^{C}$

In an initial situation $C = e^{c}$

$\therefore y-20000 = e^{-0.03t} \cdot C$ .... (4)

At $t=0$, $y(t)=0$ since initially there is no fresh air in the room,

Substituting ($y=0$) and ($t=0$) in equation (4)

$y-20000 = e^{(-0.03t)} \cdot C$

$0-20000 = e^{0} \cdot C$

$0-20000 = C$

$\therefore C = -20000$ ... (5)

Substituting eqn (5) into eqn (4)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1-e^{-0.03t})$ — (6)

Equation 6 is the model for the amount of fresh air in the room.

(b) Calculate the time at which 90% of the air in the room will become fresh.

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000$; (Since the air in the room is $20000 ft^3$ of air)

$= 18000 ft^3$

$y = 20000(1-e^{-0.03t})$

$18000 = 20000(1-e^{-0.03t})$

$0.9 = 1-e^{-0.03t}$

$e^{-0.03t} = 1-0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$= \frac{-2.303}{-0.03}$

$t = 76.77$ minutes

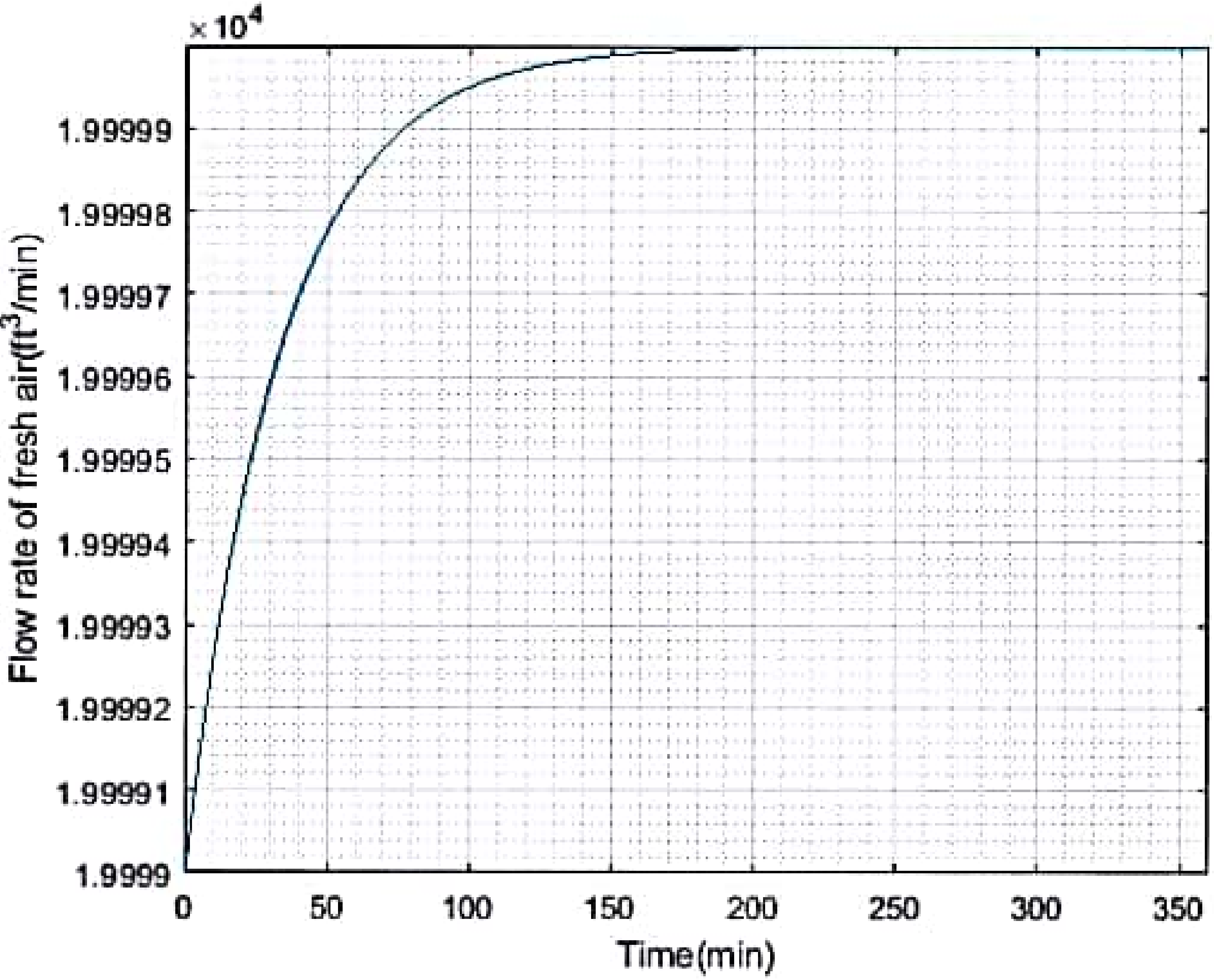
$t \approx 77$ minutes

(c) with the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ using a step time of 5 min
(Converting t from hours to minutes, 6hrs = (6×60 mins) = 360mins)

Solution

Command Window

```
Clear all
Clc
Close all
syms y
syms t
y = 20000*(1-exp(-0.03*t))
t = 0:5:360
y1 = subs(y)
plot(t,y1)
xlabel('Time (min)')
ylabel('Flowrate of fresh air (ft^3/min)')
grid on
grid minor
axis tight
```



×10^4
1.99999
1.99998
1.99997
1.99996
1.99995
1.99994
1.99993
1.99992
1.99991
1.9999
0
50
100
150
200
250
300
350
Time(min)
Flow rate of fresh air(ft^3/min)

(d) Determine the steady-state value of the amount of fresh air in the room

From the graph, it is observed that the steady-state value is $2000 ft^3$ at 215 minutes (3 hours and 35 minutes)

(e) Comment on answer in (d)

The function gives an experimental approach to the limit of air in the room as y increases with time. The steady-state value is at 215 mins. The model discussed above in a realistic situation might change because mixing can be imperfect.