

Answers TIME HELD AND ANSWERS (7) EN 403 (05) 2014

Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{dy}{dt} \rightarrow$ fresh air inflow rate $-$ fresh air outflow rate

Fresh air inflow $\rightarrow 600 \text{ ft}^3/\text{min}$

Fresh air outflow \rightarrow Note: The amount flowing out of the room is a function of the amount in room

$$\therefore \frac{600}{20000} = 0.03 \text{ min}^{-1}$$

$\therefore 0.03$ of $y(t)$ is the outflow $= 0.03y \text{ ft}^3/\text{min}$

Now:

$$\begin{aligned} \frac{dy}{dt} &= 600 - 0.03y \\ &= -0.03y + 600 \\ &= -0.03(y - 20000) \end{aligned}$$

This equation can be separated and integrated

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

Find the integral of both sides

$$\ln(y - 20000) = -0.03t + C$$
$$(y - 20000) = e^{-0.03t + C}$$

$$y - 20000 = e^{-0.03t} \cdot e^C$$

Recall $e^C = e^c =$ initial equation

$$\therefore y - 20000 = e^{-0.03t} \cdot c \quad \text{--- (1)}$$

At $t=0$, $y(t) = 0$ since the room contained no fresh air initially.

put $y=0$; $t=0$ in eqn (1)

$$y = 20000 = e^{-0.03t} \cdot c$$

$$0 - 20000 = e^0 \cdot c$$

$$0 - 20000 = \pm(c)$$

$$c = -20000 \quad \text{--- (2)}$$

put eqn (2) in eqn (1)

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000 (1 - e^{-0.03t}) \quad (3)$$

Equation (3) above is the model for the amount of fresh air in the room.

do Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \times 20,000 \text{ i.e. } 90\% \text{ of air in the room.}$$
$$= 18000 \text{ft}^3$$

$$y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{ mins} \approx \underline{\underline{77 \text{ mins}}}$$

With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hrs using a step time of 5 min

Notes $t = 6$ hrs

$$= 6 \times 60$$

$$= 360 \text{ mins}$$

Solution

Command window

Clear all

clc

close all

syms t

$$y = 20000 * (1 - \exp(-0.02 * t))$$

$$t = 0:5:360$$

yo = subs(y)

Plot(t, yo)

X label (Time (min))

Y label (Flowrate of fresh air (g^3/min))

Grid on

Grid minor

Axis tight

Output