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Computer Engineering 17/Eng 02/010

9) Let $y(t)$ be the amount of air at any time t in ft^3 in the room.

$\frac{dy}{dt} \rightarrow$ fresh air inflow rate $-$ fresh air outflow rate

fresh air inflow $\rightarrow 600 \text{ ft}^3/\text{min}$

fresh air outflow \rightarrow Note \rightarrow The amount flowing out of the room is a function of the amount in the room:

$$\therefore \frac{600}{20000} = 0.03 \text{ min}^{-1}$$

Now; i.e. 0.03 of $y(t)$ is the outflow $= 0.03y \text{ ft}^3/\text{min}$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be separated & integrated;

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

find the integral of both sides

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{(-0.03t + C)}$$

$$y - 20000 = e^{-0.03t} \cdot e^C$$

Recall $C = e^C =$ (initial equation)

$$\therefore y - 20000 = e^{-0.03t} \cdot C \quad \text{--- (1)}$$

At $t=0$, $y(t)=0$ since the room contained no fresh air initially,

Put $y=0$; $t=0$ in eqn (1)

$$y - 20000 = e^{-(0.03t)} \cdot C$$

$$0 - 20000 = e^0 \cdot C$$

$$0 - 20000 = 1(C)$$

$$C = -20000 \quad \text{--- (2)}$$

Put eqn (2) in eqn (1)

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{--- (3)}$$

eqn (3) above is the model for the amount of fresh air in the room.

$$b) \quad 90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \times 20000, \text{ i.e. } 90\% \text{ of air in the room} \\ = 18000 \text{ ft}^3$$

$$y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{ mins}$$

$$= 77 \text{ mins}$$

c) Command window

clear all

clc

close all

syms y,t

$$y = 20000 * (1 - \exp(-0.03 * t))$$

$$t = 0:5:360$$

$$y_n = \text{subs}(y)$$

Plot (t, y_n)

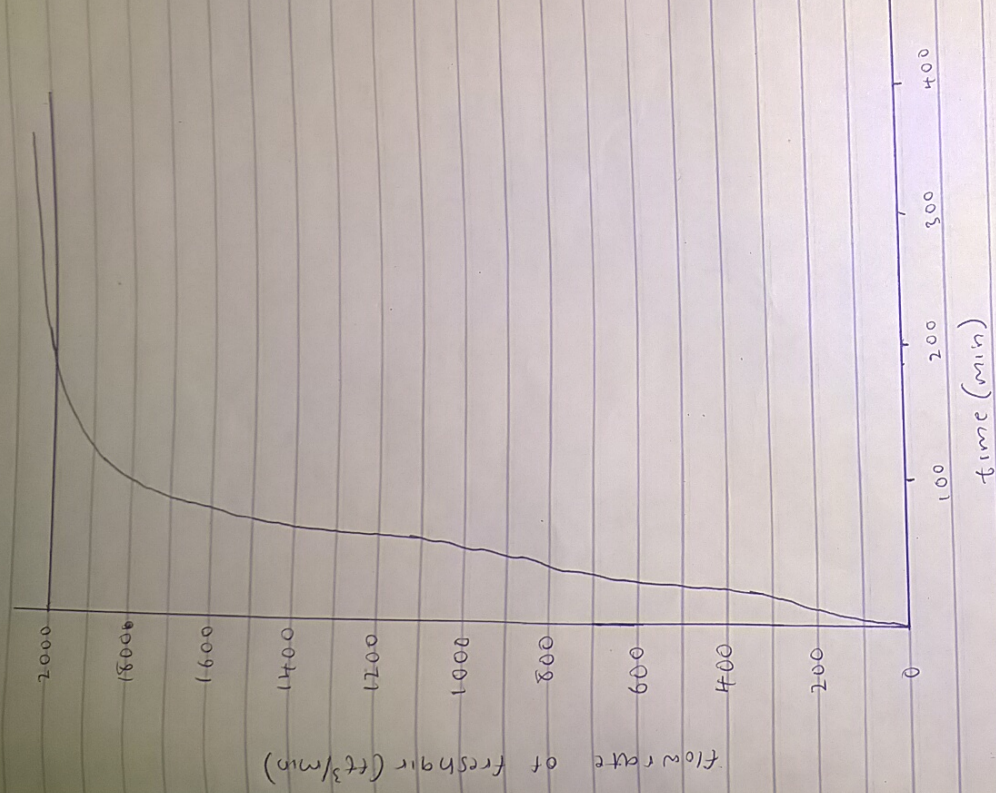
x label ('Time (min)')

y label ('flowrate of fresh air (ft³/min)')

Grid On

Grid minor

Axis tight



d) The steady-state value is 215 mins (3hr and 35 mins) of experimental approach.

e) The functions above shown an experimental approach to the limit of 2000 ft^3 as y increases with time. Also, when the steady state value approaches 2000 ft^3 at 215 minutes and continues. till 300 mins (6hrs). The model discussed becomes more realistic in pneumatic technology, although may be difficult because mixing may be imperfect