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CIVIL ENGINEERING
17/ENG03/038

A room initially contains 20,000 ft³ of fresh air flows into a room containing 20,000 ft³ of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate 600 ft³/min. If the room contains no fresh air initially

a) develop a model for the amount of fresh air in the room at any time

Answer.

Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{dy}{dt}$ = fresh air inflow rate - fresh air o

fresh air inflow $= 600 ft^3/min$

fresh air outflow Now the amount flowing out of the room is a fraction of the amount in the room

$\frac{600}{20,000} = 0.03 min$

i.e 0.03 of $y(t)$ is the outflow $= 0.03y ft^3/min$

Now

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03y (y - 20000)$

This equation can be separated & integrated

$\frac{dy}{y-20000} = -0.03dt$

find the integral of both side

$\ln(y-20000) = -0.03t + c$.

$y - 20000 = e^{(-0.03t+c)}$.

$y - 20000 = e^{-0.03t} \cdot e^c$.

Recall $C = e^c$ = initial equation

$y - 20000 = e^{-0.03t} \cdot C$

At $t = 0$, $y(t) = 0$ since the room entered no fresh air initially

put $y = 0$; $t = 0$ in equ (i)

$y - 20000 = e^{-0.03t} \cdot C$

$0 - 20000 = e^{0} \cdot C$

$0 - 20000 = 1(C)$

$C = -20000$ ..... (2)

put eqn (2) in eqn (i)

$y = 20000 - 20000 e^{-0.03t}$

$y = 20000 (1 - e^{-0.03t})$ ..... (3)

Equation 3 above is the model for the amount of fresh air in the room.

Calculate the time at which 90% of the air in the room will become fresh $90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000$ is 90% of air in the room.

$= 18000 ft^3$

$y = 20000 (1 - e^{-0.03t})$

$18000 = 20000 (1 - e^{-0.03t})$

$0.9 = 1 - e^{0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$t = \frac{\ln 0.1}{-0.03}$

$= \frac{-2.303}{-0.03}$

$= 76.77 min$

$= 77 mins$

c.) Using the app of matlab, plot the ~~diagram~~ dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6 hrs$ using a step of 5 min

Note $t = 8hrs$

$= 6 \times 60$

$= 360 mins$

Solution

```
Command window
clear all
cls
clear all
syms y, t
y = 20000 * (1 - exp(-0.03*t))
t = 0:5
yn = subs(y)
plot(t,yn)
xlabel('Time (min)')
ylabel('flowrate of fresh air (ft^3/min')
Grid on
Grid minor
Axis tight
```

output

1.) Determine the steady state value of the amount of fresh air in the room

A: The steady state value is 20000 FT^3 at 215min (3hr and 35 min) at exponential approach.

2) Comment on answer

The functions above shows an exponential approach to the limit of 20000ft^3 as y increases with time. Also, when the steady state value approaches 20000ft^3 at 215min and continues for 300min (5hrs). The model becomes more realistic in pneumatic technology, although, maybe difficult because mixing may be imperfect