CHEMICAL ENGINEERING

17/ENG01008

ENGINEERING MATHEMATICS ASSIGNMENT FOUR (4)

ENG 282

Solution

Step 1: Setting up a model

Let $F_a(t)$ denote the amount of fresh air in the room at time t.

By Balance Law:

$\frac{dF_a}{dt}$ = Fresh air inflow rate − Fresh air outflow rate

Input of fresh air = $600 ft^3/min$

Initially there was no fresh air

Hence, $F_a(0) = 0$

Also

Output of mixture = $600 ft^3/min$

mixture of fresh air and Normal air = $20000 ft^3/min$

$\frac{dF_a}{dt} = 600 - \frac{600}{20 \times 10^3} \times F_a(t)$ *

$\frac{dF_a}{dt} = 600 - 0.03F_a$

$\frac{dF_a}{dt} = -0.03(F_a - 20 \times 10^3)$

Step 2: Solution of the Model:

$$\frac{dF_{ft}}{dt} = -0.03(F_{ft} - 2\times10^3)$$

$$\frac{dF_{ft}}{dt} \times \frac{1}{F_{ft}-2\times10^3} = -0.03dt$$

to integrate both sides

$$\int \frac{dF_{ft}}{F_{ft}-2\times10^3} = \int -0.03dt$$

$$\ln F_{ft} - 2\times10^3 = -0.03t + c$$

Taking ln of both sides

$$F_{ft} - 2\times10^3 = e^{-0.03t+c}$$

$$F_{ft} = e^{-0.03t} \cdot e^c + 2\times10^3 \quad [\text{where } e^c = c]$$

$$F_{ft} = Ce^{-0.03t} + 2\times10^3 \longrightarrow \text{solution}$$

Initially there was no fresh air

Hence

$F_{ft} = 0$

Recall

$F_{ft} = 2\times10^3 + Ce^{-0.03t}$

where $t=0$, $F_{ft}=0$

$C = 2\times10^3 + C \times e^{-0.03(0)}$

$C = -2\times10^3$

Therefore substituting for C

$F_{ft(t)} = 2\times10^3 - 2\times10^3 e^{-0.03t}$ (particular solution)

(b) Time at which 70% of the air will become fresh

$$\frac{70}{100} \times 2\times10^3 = 2\times10^3 - 2\times10^3 e^{-0.03t}$$

$$1.4\times10^3 = 2\times10^3 e^{-0.03t}$$

$$-2\times10^3 = -2\times10^3 e^{-0.03t}$$

$$0.7 = e^{-0.03t}$$

$$\ln 0.7 = -0.03t$$

$$-0.3566 = -0.03t$$

$$t = 76.75 \text{ mins}$$

0.75 mins to seconds

$= 0.75 \times 60 = 45$ seconds

Therefore t = 76 minute 45 seconds

(c) 6 hours to 6 minute $= 60 \times 60 = 360$ minutes

d) Matlab m file

```
comment/uncomment
clear
clc
close all
t = 0:5:360
Ft = 2*10^3 + -2*10^3 * exp(-0.03*t)
plot(Ft, t)
xlabel('t (seconds)')
ylabel('Ft')
Grid minor
Grid on
```

(e) The steady state value of the amount of fresh air in the room $= 2\times10^3$ (ft³ of air)

f) The steady state value of the amount of fresh air in the room obtained from the equation (given) is given as a straight line where there is no longer increase in the amount of fresh air even though the

still increase in the time.

Here, the amount of fresh air in the room is steady (it does not change) with increase in time (minutes).