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PHENIX 1002

Mechatronics Engineering

2000 ft<sup>3</sup>

1. A dormitory filled with fresh air flows into a room containing 2000 ft<sup>3</sup> of air. The mixture which is made practically constant by circulating fans, is exhausted at the rate 600 ft<sup>3</sup>/min. If the room contains no fresh air initially

a) develop a model for the amount of fresh air in the room at any time  $t$ .

Answer:  
Let  $y(t)$  be the amount of air at any time  $t$  in ft<sup>3</sup> in the room.  
 $\frac{dy}{dt}$  = from air inflow rate - from air out

Fresh air inflow = 600 ft<sup>3</sup>/min

Fresh air outflow = Note the amount flowing out of the room is a fraction of the amount in the room

$$\frac{600}{20,000} = 0.03 \text{ min}^{-1}$$

$$1 - 0.03 \text{ of } y(t) \text{ is the outflow} = 0.97y(t) \text{ /min}$$

Also

$$\frac{dy}{dt} = 600 - 0.97y$$

$$= -0.97y + 600$$

$$= -0.97y (y - 2000)$$

This equation can be separated & integrated

$$\frac{dy}{y-2000} = -0.97 dt$$

find the integral of both side

$$\ln(y-2000) = -0.97t + c$$

$$y - 2000 = e^{(-0.97t + c)}$$

$$y - 2000 = e^{-0.97t} \cdot e^c$$

$$\text{when } t = 0, y = 0 = \text{initial condition}$$

$$y - 2000 = e^{-0.97t} \cdot c$$

$$t = 0, y(t) = 0 \text{ then } c = 2000$$

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Put  $y=0$ ;  $t=0$  in eqn (i)

$$y = 20000 = e^{-0.03t} \cdot C$$

$$0 - 20000 = 2^0 \cdot C$$

$$0 - 20000 = C$$

$$C = -20000 \quad \text{--- (2)}$$

Put eqn (2) in eqn (i)

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{--- (3)}$$

Equation 3 above is the model for the amount of fresh air in the room

Calculate the time at which 90% of the air in the room will be fresh.  $90\% = \frac{90}{100} = 0.9$

$$y = 0.9 \times 20000 \text{ is } 90\% \text{ of air in the room.}$$
$$= 18000 \text{ ft}^3$$

$$y = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$t = \frac{\ln 0.1}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{ min}$$

$$= 77 \text{ min}$$

$$= 77 \text{ min}$$

$$= 77 \text{ min}$$

0.) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for  $t=0$  to  $t=6$  hrs using a step of 5 min

Note  $t = 9 \text{ hrs}$

$$= 6 \times 60$$

$$= 360 \text{ min}$$

Solution

Command Window

clear all

clc

close all

syms y, t

$$y = 20000 * (1 - \exp(-0.03 * t))$$

$$t = 0.5$$

$$y_n = subs(y)$$

plot (t, y\_n)

xlabel ('Time (min)')

ylabel ('flowrate of fresh air (ft<sup>3</sup>/min)')

grid on

grid minor

axis tight

out put

d) Determine the steady state value of the amount of fresh air in the room.  
A: The steady state value is 20000 ft<sup>3</sup> at 25 min (8hr and 35 min) at exponential approach.

e) Comment on answer

The functions above shows an exponential approach to the limit of 20000 ft<sup>3</sup> as y increases with time. Also, when the steady state value approaches 20000 ft<sup>3</sup> at 25 min and continues for 30 min (6hrs). The model discussed belongs more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.