

CHINMATHUR PRUDENCE ESE
CHEMICAL ENGINEERING
17/ENG01607

It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows into a room containing 2000 ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute.

If the room contains no fresh air initially.

a) Develop a model for the amount of fresh air in the room at any time t .

b) Find the time at which 90% of the air in the room will become fresh.

c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6 \text{ hr}$ using a step time of 5 min .

d) Determine the steady state value of the amount of fresh air in the room and

e) Comment on the result obtained in (d)

Solutions.

a) Let y be the amount of air at time t in (ft^3) in the room
 $\frac{dy}{dt} = \text{Inflow rate} - \text{fresh air outflow rate}$

fresh air inflow $\rightarrow 100 \text{ ft}^3/\text{min}$

fresh air outflow $= 600$

(ABUAD), The Road to Intellectualism, Quality and Excellence

$$2000 = 0.03 \text{ min}$$

The amount flowing out of the room is a function of the amount in the room.

$$= 0.03 y \text{ ft}^3/\text{min.}$$

$$\frac{dy}{dt} = 600 - 0.03y = -0.03y + 600 = -0.03(y - 2000)$$

Equation can be resolved as

$$\frac{dy}{dt} = -0.03(y - 2000)$$

$$\int_{y-2000} \frac{dy}{y-2000} = \int -0.03 dt \cdot \ln(y-2000) = -0.03t + C$$

$$\ln(y-2000) = -0.03t + C$$

$$y - 2000 = e^{-0.03t + C}$$

$$y - 2000 = e^{-0.03t} e^C$$

←←←

At $t = 0$, $y(t) = 0$ since no fresh air was contained in the room initially.

$$y - 2000 = (e^{-0.03(0)})$$

$$0 - 2000 = C$$

$$C = -20000$$

Substituting $C = -20000$

$$y - 20000 = e^{-0.03t} - 20000$$

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t})$$

This is the model for the amount of fresh air in the room

1.) Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \text{ of } 20000$$

$$0.9 \times 20000 = 18000 \text{ ft}^3$$

$$\therefore 18000 = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9 = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{-2.3026}{-0.03}$$

$$t = 77 \text{ min.}$$

e) ~~Plotting the data~~

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* Command window

* Clear

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* $y = 20000 * (1 - \exp(-0.03 * t))$

* $t = 0.5 : 360$

* $Y_n = \text{subs}(y)$

* $\text{plot}(t, Y_n)$

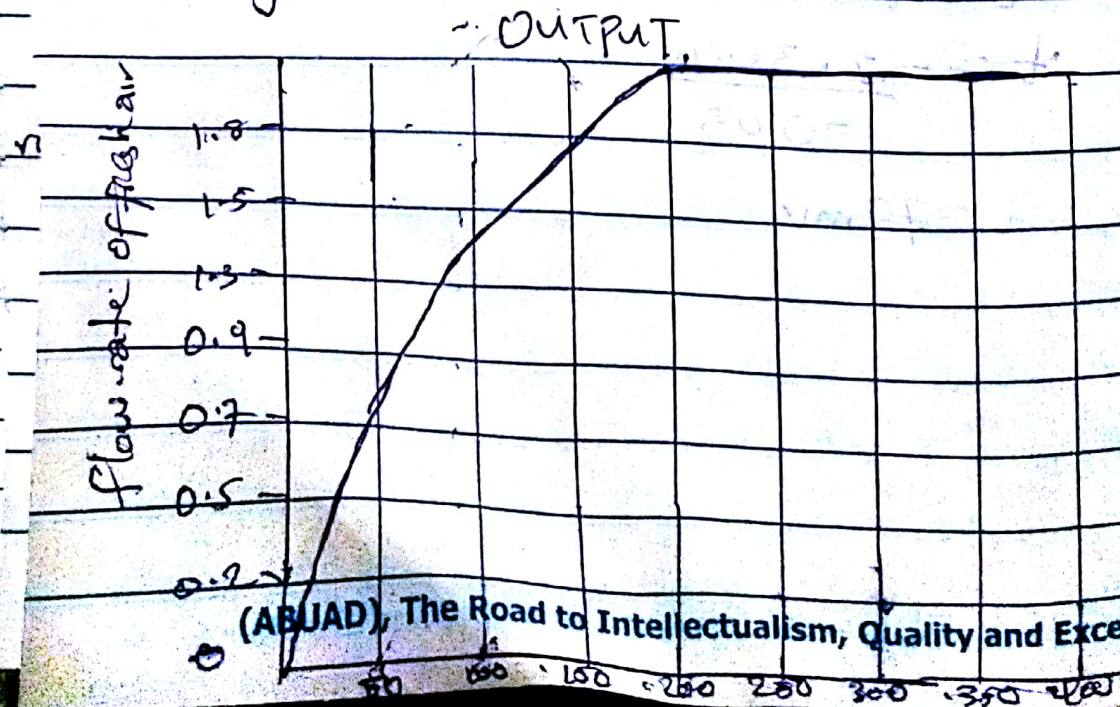
* $\text{label}('Time (min)')$

* $\text{Y label}('flowrate of fresh air')$

* grid on

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* axis tight



d) The steady state value is 20000 ft^3 of 215 min of the exponential approach

e) The steady state value proves the function which shows the exponential growth of limit of 20000 ft^3 as y increases with time.