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Mechanical Engineering
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a) Let y be the amount of air at time t (ft^3) in room

$\frac{dy}{dt}$ = air in flow rate − fresh air out flow rate

fresh air inflow → $100 ft^3/min$

fresh air outflow → $\frac{600}{2000} = 0.03 min$

The amount flowing out of the room $f(t) = 0.03y ft^3/min$

$$\frac{dy}{dt} = 600 - 0.03y$$
$$= -0.03y + 600$$
$$= -0.03(y - 20000)$$

Equation can be resolved as

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\frac{dy}{(y-20000)} = -0.03dt$$

$$\ln(y - 20000) = -0.03t + c$$
$$\ln(y - 20000) = -0.03t + c$$
$$\Rightarrow y - 20000 = e^{-0.03t + c}$$
$$\Rightarrow y - 20000 = e^{-0.03t} e^{c}$$

$C = e^{c}$ = initial condition

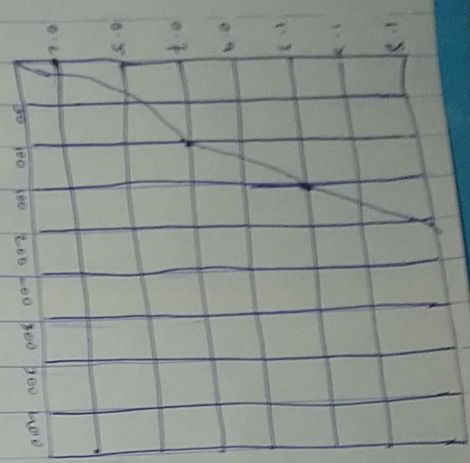
$$y - 20000 = e^{-0.03t} \cdot C$$

At $t = 0$

$$y - 20000 = Ce^{-0.03(0)}$$
$$0 - 20000 = C$$
$$C = -20000$$

$$y - 20000 = e^{-0.03t} - 20000$$
$$y = (e^{-0.03t} - 20000) + 20000$$
$$y = 20000 - 20000e^{-0.03t}$$
$$y = 20000(1 - e^{-0.03t})$$

b) $90\% = \frac{90}{100} = 0.9$



$y = 0.90 \times 20000$

$= 0.9 \times 20000 = 18000 \, 13$

$18000 = 20000 (1 - e^{0.05t})$

$0.9 = 1 - e^{-0.05t}$

$e^{-0.05t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{2.3026}{0.03} = 72 mins$

matlab program

→ command window

clear

clc

close all

syms t

y = 2000*(1.exp(0.07*t))

t = 0:5:360

yn = subs(y)

plot(t, yn)

label('time(min)')

"ylabel('flowrate of fresh air)

grid on

grid minor

axis tight

d) The steady state value is 2000 ft^3 at 215min of the exponential approach

e) The function shows an exponential growth approach to the Limit of 2000 ft^3 as y increase with time and the steady value was 2000 ft^3 at 20 min. It worked for 6 hours