

ANSWERS

Let $y(t)$ be the amount of air at any time t in ft^3 in the room.

$\frac{dy}{dt} \rightarrow$ fresh air inflow rate - fresh air outflow rate

Fresh air inflow = $600 \text{ ft}^3/\text{min}$

Fresh air outflow = Note: The amount flowing out of the room is a function of the amount in room.

$$\therefore \frac{600}{20000} = 0.03 \text{ min}^{-1}$$

i.e. 0.03 of $y(t)$ is the outflow = $0.03y \text{ ft}^3/\text{min}$.

Now;

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

This equation can be separated and integrated

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

Find the integral of both sides

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

Recall $C = e^C = \text{initial equation}$

$$\therefore y - 20000 = e^{-0.03t} \cdot C \quad \text{--- (1)}$$

At $t=0$, $y(t)=0$ since the room contained no fresh air initially.

put $y=0$; $t=0$ in eqn (1)

$$y = 20000 = e^{-(0.03 \cdot 0)} \cdot C$$

$$0 - 20000 = e^0 \cdot C$$

$$0 - 20000 = 1(C)$$

$$C = -20000 \quad \text{--- (2)}$$

Put eqn (2) in eqn (1)

$$y = 20000 - 20000 e^{-0.03t}$$
$$y = 20000 (1 - e^{-0.03t}) \quad (3)$$

Equation (3) above is the model for the amount of fresh air in the room.

do Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \times 20,000 \text{ i.e. } 90\% \text{ of air in the room.}$$

$$= 18000 \text{ ft}^3$$

$$y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{ mins} \approx 77 \text{ mins.}$$

$$= 76.77 \text{ mins} \approx 77 \text{ mins.}$$

do

Work the end of MATHE, plot the dynamic response of the amount of fresh air in the room for $T=0$ to $T=60$ using a step time of 5 min

$$\text{Note: } T = 6 \text{ hrs}$$

$$= 6 \times 60$$

$$= 360 \text{ mins}$$

Solution

Command window

clear all

clc

close all

syms y,t

$$y = 20000 * (1 - \exp(-0.03 * t))$$

$$T = 0.5 : 360$$

yn = subs(y)

Plot (t, yn)

X Label (Time (min))

Y Label (Flowrate of fresh air (l³/min))

Grid on;

Grid minor

Axis tight.

Output