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17/ENG06/044

MECHANICAL
ENGINTEERING

(a) Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{dy}{dt}$ \rightarrow fresh air inflow rate - fresh air outflow rate

Fresh air inflow $\rightarrow 600 \text{ ft}^3/\text{min}$

Fresh air outflow \rightarrow Note: The amount flowing out of the room is a function of the amount in the room

$$600 - \frac{y}{200} \times 0.03$$

i.e. 0.03 of $y(t)$ is the outflow $= 0.03 \frac{y}{200} \text{ ft}^3/\text{min}$

Now:

$$\begin{aligned} \frac{dy}{dt} &= 600 - 0.03y \\ &= -0.03y + 600 \\ &= -0.03(y - 20000) \end{aligned}$$

This equation can be separated and integrated

$$\left(\frac{dy}{y - 20000} \right)$$

Find the integral of both sides

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{(-0.03t + C)}$$

$$y - 20000 = e^{-0.03t} \cdot e^C$$

Recall $C = e^C = \text{arbitrary equation}$

$$\therefore y - 20000 = e^{-0.03t} \cdot C$$

At $t = 0$, $y(t) = 0$ since the room contained no

fresh air initially

put $y = 0$; $t = 0$ in eqn (1)

$$y - 20000 = e^{-(0.03t)} \cdot C$$

$$0 - 20000 = e^0 \cdot C$$

$$0 - 20000 = 1 \cdot C$$

$$C = -20000 \quad \text{--- (2)}$$

Put eqn (2) in eqn (1)

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad \text{--- (3)}$$

Equation (3) above is the model for the amount of

$$\textcircled{b} \quad 90\% = \frac{90}{100} = 0.9$$

$$\therefore y = 0.9 \times 20,000 = 18,000 \text{ of air in the room}$$

$$y = 18000 e^{-0.03t}$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-0.03}{-0.03}$$

$$= 2.803$$

$$= 0.03$$

$$= 76.77 \text{ mins}$$

$$\approx 77 \text{ mins}$$

\textcircled{c} command window

clear all

clc

close all

syms y, t

$$y = 20000 * (1 - \exp(0.03 * t))$$

$$t = 0.5 : 360$$

$$y_n = \text{subs}(y)$$

plot(t, y_n)

Xlabel('Time (min)')

Ylabel('Temperature of fresh air (F)')/m

grid on

grid

Axes ^{minor} light

Output

2000

1800

1600

1400

1200

1000

800

600

400

200

0

100

200 & 300

(time in min)

400

- ① The steady state value is 2000 μs at 2.75 mm (3 hr and 35 mins) up is experimental approach.