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 Computer Engineering

Let  $y(t)$  be the amount of air at any time  $t$  in  $\text{ft}^3$  in the room.

$\frac{dy}{dt} \rightarrow$  fresh air inflow rate - outflow rate

Fresh air inflow =  $600 \text{ ft}^3/\text{min}$

" " outflow =  $18 \text{ ft}^3/\text{min}$  - The amount

flowing out of the room is a function of the amount in the room.

$$\frac{600}{20000} = 0.03 \text{ min}^{-1}$$

Now,  $\frac{dy}{dt} = 600 - 0.03y$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

$$= -0.03 dt$$

$$\frac{dy}{y - 20000}$$

$$\ln|y - 20000| = 0.03t + C$$

$$y - 20000 = e^{0.03t + C}$$

Recall  $C = e^C = \text{initial equation}$

$$y - 20000 = e^{-0.03t} \cdot C$$

At  $t=0$ ,  $y(t) = 0$

$$y - 20000 = C e^{-0.03(0)}$$

$$0 - 20000 = C$$

$$C = -20000$$

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000 (1 - e^{-0.03t})$$

model of fresh air in the room

b. time when 90% of the air will be fresh

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \text{ of } 20000 \\ = 0.9 \times 20000 \\ = 18000 \text{ ft}^3$$

$$\therefore y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln 0.1$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$t = 76.47$$

$$t = 77 \text{ mins}$$

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c) cmd of matlab plot the dynamic response

$$t = 6 \text{ hrs}$$

$$= 6 \times 60$$

$$= 360 \text{ mins}$$

System

Command window

Clear all

Cls

Close all

Signs y,t,r

$$y = 20000 * (1 - \exp(-0.03*t))$$

$$t = 0:5:360$$

y = subs(y)

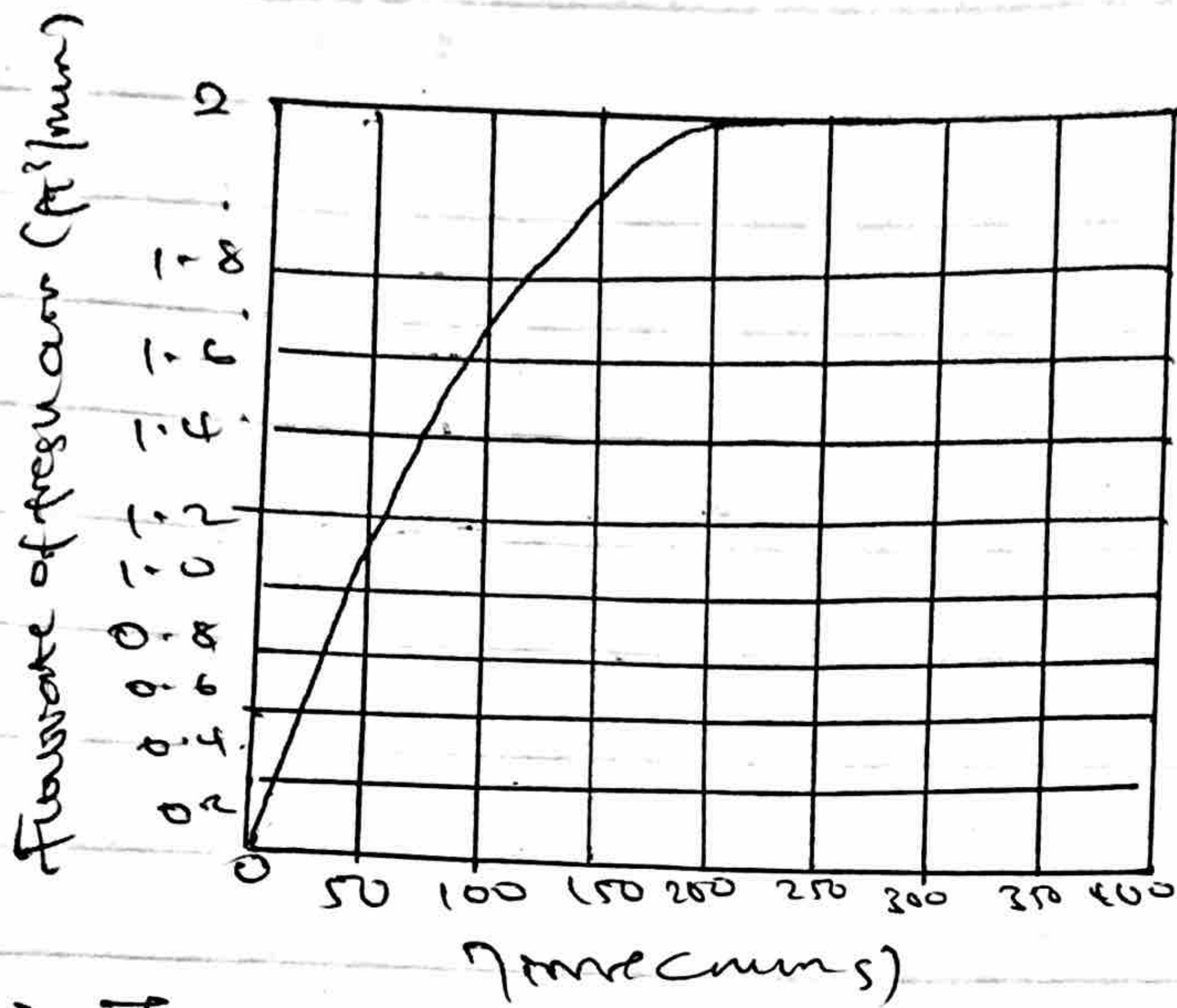
X label ('Time (min)')

Y label ('fraction of fresh air (ft<sup>3</sup>/min)')

Grid on

Grid minor

AXES right



d. A: The steady-state value is  $2000 \text{ ft}^3$  at 215 mins

c. The function shows an exponential approach to the limit of  $2000 \text{ ft}^3$  as  $y$  increases with time. Also when the steady flow state value approaches  $2000 \text{ ft}^3$  at 215 mins and moves to 360 mins i.e. 6 hours. The model becomes more realistic pneumatic technology wise, although it may be difficult because mixing may be imperfect.