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17/mbbs05/010
Biomedical Engineering

Answers

Let $y(t)$ be the amount of air at any time t in ft^3 in the room.

$\frac{dy}{dt}$ → fresh air inflow rate − fresh air outflow rate

Fresh air inflow − $600 ft^3/min$

Fresh air outflow − Note: The amount flowing out of the room is a function of the amount in room.

$\therefore \frac{600}{20000} = 0.03\ min$

i.e 0.03 of $y(t)$ is the outflow $= 0.03y\ ft^3/min$.

Now;

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

This equation can be separated and integrated

$\frac{d}{(y - 20000)} = -0.03\,dt$

Find the integral of both sides

$\ln(y - 20000) = -0.03t + c$

$y - 20000 = e^{(-0.03t + c)}$

$y - 20000 = e^{-0.03t} = e^c$

Recall $c = e^c$ = initial equation

$\therefore y = 20000 = e^{-0.03t}; c$ ——— (1)

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially.

put $y = 0$; $t = 0$ in eqn (1)

$y = 20000 = e^{-(0.03t)}c$

$0 - 20000 = e^0 \cdot c$

put eqn (2) in eqn (1)

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000\left(1 - e^{-0.03t}\right) \quad \text{---- (3)}$$

Equation (3) above is the model for the amount of fresh air in the room.

(b) Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$y = 0.9 \times 20,000$ i.e 90% of air in the room.

$$= 18000 ft^3$$

$$y = 20000\left(1 - e^{-0.03t}\right)$$

$$18000 = 20000\left(1 - e^{-0.03t}\right)$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{ mins} \simeq 77 \text{ mins}$$

(c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6hr$ using a step time of 5 min

Note: $t = 6hrs$

$= 6 \times 60$

$= 360 mins$

Solution

Command window.

```
clear all
clc
close all
syms y,t
y = 20000 * (1 - exp(0.03 * t))
t = 0.5 : 360
yn = subs(y)
Plot(t, yn)
X label (Time (min))
Y label (Flowrate of fresh air (ft^3/min))
Grid on.
Grid minor
Axis tight.
```

Output