

- It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows into a room containing $20,000 \text{ ft}^3$ of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of $600 \text{ ft}^3/\text{min}$. If the room contains no fresh air initially

a) Develop a model for the amount of fresh air in the room at any time t .

Solution:

Let $y(t)$ be the amount of air at any time (t) in ft^3 in the room

$$\frac{dy}{dt} \Rightarrow \text{fresh air inflow rate} - \text{Fresh air outflow rate}$$

$$\frac{dy}{dt} \Rightarrow \text{Change in Flow Rate}$$

$$\text{fresh air inflow} \Rightarrow 600 \text{ ft}^3/\text{min}$$

$$\therefore \frac{600}{20,000} = 0.03 \text{ min}^{-1}$$

i.e. 0.03 of $y(t)$ is the outflow $= 0.03y \text{ ft}^3/\text{min}$

$$\therefore \frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20,000)$$

Thus equation can be separated and integrated;

$$\frac{dy}{(y - 20,000)} = -0.03 dt$$

find the integral of both sides.

$$\ln(y - 20,000) = -0.03t + C$$

$$y - 20,000 = e^{(-0.03t + C)}$$

$$y - 20,000 = e^{-0.03t} \cdot e^C$$

Recall $C = e^C = \text{Initial Equation}$

$$\therefore y - 20,000 = e^{-0.03t} \cdot C \quad \dots (1)$$

At $t=0$, $y(t)=0$ since the room contains no fresh air initially

put $y=0$; $t=0$ into eqn (1)

$$y = 20,000 = e^{-0.03t} \cdot C$$

$$0 - 20,000 = e^{-0.03(0)} \cdot C$$

$$-20,000 = e^0 \cdot C$$

$$C = -20,000 \quad \dots (2)$$

Put eqn (2) into (1)

$$y = 20,000 - 20,000 e^{-0.03t}$$

$$y = 20,000 (1 - e^{-0.03t}) \quad \dots (3)$$

Equation (3) is the model for the amount of fresh air in the room.

b) Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \times 20,000 \quad \text{i.e. } 90\% \text{ of air in the room}$$
$$= 18,000 \text{ ft}^3$$

$$y = 20,000 (1 - e^{-0.03t})$$

$$18,000 = 20,000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$t = 77 \text{ mins}$$

c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=60$ using a step of 5 min.

- Command window
- clear
- clc
- close all
- Syms y, t
- $y = 20,000 * [1 - \exp(-0.03 * t)]$
- $t = 0:5:360$
- $yn = \text{subs}(y)$
- plot(t, yn)
- X label ('Time (min)')
- Y label ('Flow rate of fresh air (ft³/min)')
- Grid on
- Grid minor
- Axis tight.

Output : [MATLAB DISPLAY].

d) Determine the steady state value of the amount of fresh air in the room.

Solution

The steady state value is 20,000 ft³/at 215 mins (3hrs and 35mins) at exponential approach.

e) Comment:

The functions above shows an exponential equation to the limit of 20,000 ft³ as y increases with time. Also, when the steady state value approaches 20,000 ft³ at 215 mins and continues till 300 min (6 hrs). The model discussed becomes more real in gas (pneumatic technology), although tough because mixing may be imperfect.