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It is discovered that of fresh air flows into a room containing 20000ft of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate $600 ft^3/min$. If the room contains no fresh initially
a) develop a model for the amount of fresh air in the room at any time]

Answer.
Let $y(t)$ be the amount of air at any time t in ft^3 in the room

$\frac{dy}{dt}$ = fresh air inflow rate - fresh air o

fresh air inflow = $600 ft^3/min$
fresh air outflow Note the amount flowing out of the room is a fraction of the amount in the room

$\frac{600}{20,000} = 0.03 min$

i.e 0.03 of $y(t)$ is the outflow $= 0.03y ft^3/min$

Now

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$
$= -0.03y(y - 20000)$

This equation can be separated & integrated

$\frac{dy}{y-20000} = -0.03dt$

find the integral of both side

$\ln(y-20000) = -0.03t + c$
$y - 20000 = e^{(-0.03t + c)}$
$y - 20000 = e^{-0.03t} \cdot e^{c}$
Recall, $C = e^{c}$ = initial equation
$y - 20000 = e^{-0.03t} C$

At $t = 0$, $y(t) = 0$ since the room contained no fresh initially

put $y = 0$; $t = 0$ in equ (1)

$y = 20000 = e^{-0.03t} \cdot c$

$0 - 20000 = e^{0} \cdot c$

$0 - 20000 = 1(c)$

$c = -20000$ ..... (2)

put eqn (2) in eqn (i)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ..... (3)

Equation 3 above is the model for the amount of fresh air in the room.

Calculate the time at which 90% of the air in the room will become fresh $90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000$ is 90% of air in the room.

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$t = \frac{\ln 0.1}{-0.03}$

$= \frac{-2.303}{-0.03}$

$= 76.77 min$

$= 77 mins$

Q.) Write the air of matlab, plot the ~~impulse~~ dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6 hrs$ using a step of 5 min

Note $t = 6hrs$

$= 6 \times 60$

$= 800 mins$

Solution

Command window

clear all

clc

Close all

Syms y, t

$y = 20000 * (1 - \exp(-0.03^{*} t))$

$t = 0.5$

$y_n = \text{subs}(y)$

plot (t, y_n)

xlabel ('Time (min)')

Ylabel ('flowrate of fresh air $(ft^3/min$

Grid on

Grid minor

Axis tight

output

1-) Determine the steady state value of the amount of fresh air in the room.

A : The steady state value is 20000 Ft^3 at 215min (3hr and 35 min) at exponential approach.

2-) Comment on answer

The functions above shows an exponential approach to the limit of $20000 ft^3$ as y increases with time. Also, when the steady state value approaches $20000 ft^3$ at 215min and continues for 300min (6hrs). The model becomes more realistic in practical technology, although, maybe difficult because mixing may be imperfect.