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9) Let $y(t)$ be the amount of air at any time $t$ in $ft^3$ in the room.

$\frac{dy}{dt}$ → fresh air inflow rate − fresh air airflow rate

fresh air inflow → $600 ft^2/min$

fresh air outflow → Note → The amount flowing out of the room is a function of the amount in the room.

$\therefore \frac{600}{20000} = 0.03\ min$

i.e 0.03 of $y(t)$ is the outflow = $0.03y ft^3/min$

Now;

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

This equation can be seperated & integrated;

$\frac{dy}{(y-20000)} = -0.03\,dt$

find the integral of both sides

$\ln(y-20000) = -0.03t + C$

$y - 20000 = e^{(-0.03t+C)}$

$y - 20000 = e^{-0.03t} \cdot e^{C}$

Recall $C = e^{C}$ = initial equation

$\therefore y - 20000 = e^{-0.03t} \cdot C$ ——— ①

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially,

Put $y = 0$ ; $t = 0$ in eqn ①

$y - 20000 = e^{-(0.03t)} \cdot C$

$0 - 20000 = e^{0} \cdot C$

$0 - 20000 = 1(C)$

$C = -20000$ ——— ②

Put eqn (2) int eqn (1)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ——— ③

eqn ③ above is the model for the amount of fresh air in the room.

b) 90% = $\frac{90}{100}$ = 0.9

$y = 0.9 \times 20000$ , ie 90% of air in the room

$= 18000 \text{ ft}^3$

$y = 20000 (1 - e^{-0.03t})$

$18000 = 20000 (1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03t}$

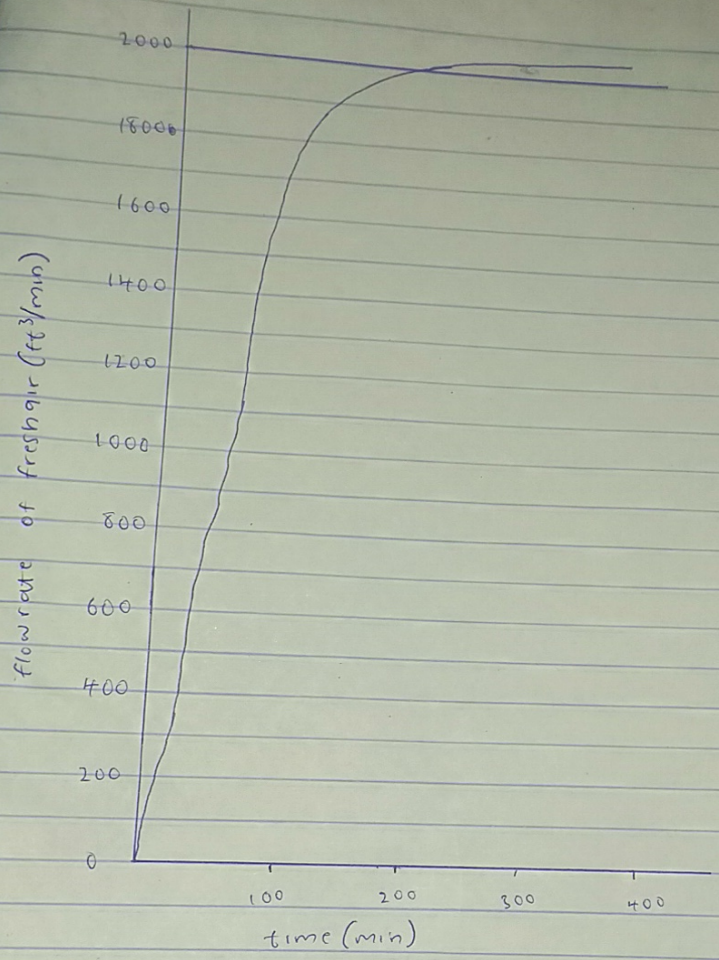
$= \frac{-2.303}{-0.03}$

$= 76.77 \text{ mins}$

$= 77 \text{ mins}$

c) Command window

```
clear all
Clc
Close all
Syms y,t
y = 20000 * (1-exp(-0.03*t))
t = 0:5:360
yn = subs(y)
Plot (t, yn)
X label ('Time (min)')
Y label ('flowrate of fresh air (ft^3/min)')
Grid On
Grid minor
Axis tight
```




d) The steady-state value is 215 mins (3hr and 35 mins) of experimental approach.

e) The functions above shown an experimental approach to the limit of 20000ft$^3$ as y increases with time. Also, when the steady state value approaches 20000ft$^2$ at 215 minutes and continues. till 300 mins (6 hrs). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect