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17/ENG04/1014.
ELECT-ELECT
MATH ASSIGNMENT 4

It is discovered that 600 ft³/min of fresh air flows into a room containing 20,000 ft³ of air. The mixture which is made practically uniform by circulating fans is exhausted at the rate of 600 ft³/min. if the room contains no fresh air initially @ develop a model for the amount of fresh air in the room at any time (t).

Solution.

@ Let y(t) be the amount of air at time (t) in the room

Change in fresh air = fresh air inflow rate - fresh air outflow rate.

∴ y' = fresh air inflow rate - fresh air outflow rate.

fresh air inflow = 600 ft³/min

fresh air outflow is a function of the amount in the room.

$$\therefore \frac{600}{20,000} = 0.03 \text{ of } y(t) = 0.03y \text{ ft}^3/\text{min}$$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$\frac{dy}{dt} = -0.03y + 600$$

$$\therefore \frac{dy}{dt} = -0.03[y - 20000]$$

$$\therefore \frac{dy}{[y - 20000]} = -0.03\,dt.$$

Integrating both sides-

$$\int \frac{dy}{[y - 20000]} = -0.03 \int dt$$

$$\therefore \ln[y - 20000] = -0.03t + c$$

$$\therefore y - 20000 = e^{[-0.03t + c]}$$

$$\therefore y - 20000 = e^{-0.03t} \cdot e^c \text{ [where } e^c = c\text{]}$$

$$y - 20000 = ce^{-0.03t} \text{ [General solution]}$$

At $t = 0$, $y(t) = 0$ Since the room contained no fresh air initially

$\therefore 0 - 20000 = Ce^{-0.03[0]}$

$-20000 = C \cdot e^{0}$

$-20000 = C$

$\therefore C = -20000$

$\therefore y = 20000 - 20000e^{-0.03t}$ [Particular Solution].

This is the model for the amount of fresh air in the room $y = 20000[1 - e^{-0.03t}]$.

(b) Calculate the time at which 90% of the air in the room will become fresh

$\therefore 90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20000 = 18000 ft^3$

Recall $y = 20000[1 - e^{-0.03t}]$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$\ln(0.1) = -0.03t$

$t = \frac{-2.303}{-0.03}$

$t = 76.77 \text{ mins} \simeq 77 \text{ mins}$

(c) With the aid of MATLAB, Plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hours using a step of 5 min

$\therefore 6 \text{ hours} = 6 \times 60 = 360 \text{ mins}$

Solution

```
Command window
clear all
clc
close all
Syms t
y = 2000 * (1 - exp(-0.03*t))
```

```
t = 0:5: 360
y = 5468 (y)
plot (t, y)
xlabel ("time (mins)")
ylabel ("flow rate of fresh air (ft^3/min)")
grid on
grid minor
axis tight
```

(d) Determine the steady-state value of the amount of fresh air in the room

- The steady state value is 20000 ft^3 at 25 mins of estimated approach

(e) Comment on result in (d)

The function shows an exponential approach to the limit of 20000 ft^3 as y increases with time. when the steady state value approaches 20000 ft^3 at 215 mins and continues till 360 mins the model becomes more reliable in pharmaceutical technology maybe difficult because mixing may be interfered