Name: Ikebulo Delight Nwemjika
Dept : Mechanical Engineering
Matno: 17/Engo61043

1.

— It is discovered that 600 ft^3/min of fresh air flows into a room containing 20,000 ft^3 litre of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of 600 ft^3/min. If the room contains no fresh air initially.

a) Develop a model for the amount of fresh air in the room at any time t.

Solution

Let $y(t)$ be the amount of air at any time (t) in ft^3 in the room

$\frac{dy}{dt}$ ⇒ fresh air inflow rate − Fresh air outflow rate.

$\frac{dy}{dt}$ ⇒ Change In Flow Rate

fresh air inflow ⇒ 600 ft^3/min

$$\therefore \frac{600}{20{,}000} = 0.03 \text{ min}$$

i.e 0.03 of $y(t)$ is the outflow = $0.03y$ ft^3/min

$$\therefore \frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20{,}000)$$

Thus equation can be seperated and integrated;

$$\frac{dy}{(y-20{,}000)} = -0.03\,dt$$

find the Integral of both sides.

$$\ln(y - 20{,}000) = -0.03t + C$$

$$y - 20{,}000 = e^{(-0.03t + C)}$$

$$y - 20{,}000 = e^{-0.03t} \cdot e^{c}$$

Recall $C = e^{c}$ = Initial Equation.

$$\therefore y - 20{,}000 = e^{-0.03t} \cdot C \quad \cdots (1)$$

At $t = 0$, $y(t) = 0$ since the room contain no fresh air initially

put $y = 0$; $t = 0$ into eqn (1)

$$y - 20,000 = e^{-0.03t} \cdot C$$

$$0 - 20,000 = e^{-0.03(0)} \cdot C$$

$$-20,000 = e^{0} \cdot C$$

$$C = -20,000 \quad \ldots (2)$$

Put eqn (2) into (1)

$$y = 20,000 - 20,000^{-0.03t}$$

$$y = 20,000(1 - e^{-0.03t}) \quad \ldots (3)$$

Equation (3) is the model for the amount of fresh air in the room.

b) Calculate the time at which 90% of the air in the room will become fresh.

$$90\% = \frac{90}{100} = 0.9$$

$y = 0.9 \times 20,000$ i.e 90% of air in the room

$= 18000 \text{ ft}^3$

$$y = 20,000(1 - e^{-0.03t})$$

$$18000 = 20,000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$t = 77 \text{ mins}$$

c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hours using a step of 5 min.

- Command window

```
- clear
- clc
- close all
- Syms y, t
- y = 20,000 * [1 - exp(-0.03*t)]
- t = 0:5:360
- yn = subs (y)
- plot (t, yn)
- X Label ('Time (min)')
- Y Label ('Flow rate of fresh air (ft^3/min)')
- Grid on
- Grid minor
- Axis tight.
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Output : [MATLAB DISPLAY].

d) Determine the steady state value of the amount of fresh air in the room.

Solution

The steady state value is 20,000 ft^3/ at 215 mins (3hrs and 35mins) at exponential approach.

e) Comment:

The functions above shows an exponential equation to the limit of 20,000 ft^3 as y increases with time. Also, when the steady state value approaches 20,000ft^3 at 215 mins and continues till 300 min (6hrs). The model discussed becomes more real in gas (pneumatic technology), although tough because mixing may be imperfect.