Bullem, Florence Ilwebochuwch

16/ENG01/005

Chemical Engineering

ENG 382 - Engineering mathematics IV

$d = \alpha \beta^t$

$\ln d = \ln \alpha + t \ln \beta$ —— (1)

Straight line Equation, $y = a_0 + a_1 x$ —— (2)

comparing (1) and (2)

$a_0 = \ln \alpha$ = intercept

$a_1 = \ln \beta$ = slope

$\ln d = \ln \alpha + t \ln \beta$; $\ln d = \ln \alpha \Sigma N + \ln \beta \Sigma t$

$t \ln d = t \ln \alpha + t^2 \ln \beta$; $t \ln d = \ln \alpha \Sigma t + \ln \beta \Sigma t^2$

$$\begin{bmatrix} \Sigma N & \Sigma t \\ \Sigma t & \Sigma t^2 \end{bmatrix} \begin{bmatrix} \ln \alpha \\ \ln \beta \end{bmatrix} = \begin{bmatrix} \Sigma \ln d \\ \Sigma t \ln d \end{bmatrix}$$

t(hr)	d(m)	ln d	t ln d	t^2
0	2	0.693147	0	0
1	5	1.609438	1.60943791	1
2	19	2.944439	5.88887796	4
3	50	3.912023	11.7360690	9
4	151	5.017280	20.0691193	16
5	470	6.152733	30.7636635	25
6	1435	7.268920	43.6135208	36
7	4512	8.414496	58.9014706	49
8	12936	9.467769	75.7421552	64
9	41125	10.62437	95.6193434	81
10	111021	11.61747	116.174747	100
$\Sigma = 55$		$\Sigma = 67.72209$	$\Sigma = 460.1184$	$\Sigma = 385$

$$\begin{bmatrix} 11 & 55 \\ 55 & 385 \end{bmatrix} \begin{bmatrix} \ln\alpha \\ \ln\beta \end{bmatrix} = \begin{bmatrix} 67.72209 \\ 460.11840 \end{bmatrix}$$

$$AX = B$$

$$\therefore X = A^{-1}B$$

$$A^{-1} = \frac{1}{(11\times 385)-(55\times 55)} \begin{bmatrix} 385 & -55 \\ -55 & 11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1210} \begin{bmatrix} 385 & -55 \\ -55 & 11 \end{bmatrix}$$

$$X = \frac{1}{1210} \begin{bmatrix} 385 & -55 \\ -55 & 11 \end{bmatrix} \begin{bmatrix} 67.72209 \\ 460.11840 \end{bmatrix}$$

$$X = \frac{1}{1210} \begin{bmatrix} (385\times 67.72209) + (-55\times 460.11840) \\ (-55\times 67.72209) + (11\times 460.11840) \end{bmatrix}$$

$$X = \frac{1}{1210} \begin{bmatrix} 766.49265 \\ 1336.58745 \end{bmatrix}$$

$$\begin{bmatrix} \ln\alpha \\ \ln\beta \end{bmatrix} = \begin{bmatrix} 0.633465 \\ 1.104618 \end{bmatrix}$$

$\ln\alpha = 0.633465$

$\ln\beta = 1.104618$

The equation becomes, $\ln d = 0.633465 + 1.104618t$

~~(k)~~ hence, $\alpha = e^{0.633465}$

$\alpha = 1.884128$

$\beta = e^{1.104618}$

$\beta = 3.018071$

$y = 1.884128 \times 3.018071^{t}$

t(hr)	d(m)	ln (d)	ln(d) sim	ln(d) sim error	ln(d) mean error	[ln(d) sim error]^2	[ln(d) mean error]^2
0	2	0.693147	0.633465	-5.523089	-5.463407	30.504512	29.848816
1	5	1.609434	1.738083	-4.418471	-4.547116	19.522886	20.676264
2	19	2.944439	2.842701	-3.313853	-3.212115	10.981622	10.317683
3	50	3.912023	3.947318	-2.209235	-2.244531	4.880719	5.037919
4	151	5.01728	5.051936	-1.104618	-1.139274	1.220181	1.297945
5	470	6.152733	6.156554	0	-0.003821	0	0.000046
6	1435	7.268920	7.261171	1.104618	1.112366	1.220181	1.237358
7	4512	8.414496	8.365789	2.209235	2.257942	4.880719	5.098302
8	12936	9.467769	9.470407	3.313853	3.311216	10.981622	10.964151
9	41125	10.62437	10.57502	4.418471	4.467818	19.522886	19.961398
10	111021	11.61747	11.67964	5.523089	5.460921	30.504512	29.821658

ln(d) sim

when $t=0$; $\ln d = 0.633465 + 1.104618(0) = 0.633465$

$t=1$; $\ln d = 0.633465 + 1.104618(1) = 1.738083$

$t=2$; $\ln d = 0.633465 + 1.104618(2) = 2.842701$

$t=3$; $\ln d = 0.633465 + 1.104618(3) = 3.947318$

$t=4$; $\ln d = 0.633465 + 1.104618(4) = 5.051936$

$t=5$; $\ln d = 0.633465 + 1.104618(5) = 6.156554$

$t=6$; $\ln d = 0.633465 + 1.104618(6) = 7.261171$

$t=7$; $\ln d = 0.633465 + 1.104618(7) = 8.365789$

$t=8$; $\ln d = 0.633465 + 1.104618(8) = 9.470407$

$t=9$; $\ln d = 0.633465 + 1.104618(9) = 10.57502$

$t=10$; $\ln d = 0.633465 + 1.104618(10) = 11.67964$

$$\ln(d)\text{ mean} = \frac{(0.693147 + 1.609434 + 2.944439 + 3.912023 + 5.01728 + 6.152733 + 7.268920 + 8.414496 + 9.467769 + 10.62437 + 11.61747)}{11}$$

$\ln(d)$ mean $= 6.156554$

$\ln(d)_{sim\,error} = \ln(d)_{sim} - \ln(d)_{mean}$

when $t=0$ $\ln(d)_{sim\,error} = 0.633465 - 6.156554 = -5.523089$

$t=1$; $\ln(d)_{sim\,error} = 1.738083 - 6.156554 = -4.418471$

$t=2$; $\ln(d)_{sim\,error} = 2.542701 - 6.156554 = -3.313853$

$t=3$; $\ln(d)_{sim\,error} = 3.947318 - 6.156554 = -2.209235$

$t=4$; $\ln(d)_{sim\,error} = 5.051936 - 6.156554 = -1.104618$

$t=5$; $\ln(d)_{sim\,error} = 6.156554 - 6.156554 = 0$

$t=6$; $\ln(d)_{sim\,error} = 7.261171 - 6.156554 = 1.104618$

$t=7$; $\ln(d)_{sim\,error} = 8.365789 - 6.156554 = 2.209235$

$t=8$; $\ln(d)_{sim\,error} = 9.470407 - 6.156554 = 3.313853$

$t=9$; $\ln(d)_{sim\,error} = 10.57502 - 6.156554 = 4.418471$

$t=10$; $\ln(d)_{sim\,error} = 11.67964 - 6.156554 = 5.523089$

$\ln(d)_{merror} = \ln(d) - \ln(d)_{mean}$

$t=0$; $\ln(d)_{merror} = 0.693147 - 6.156554 = -5.463407$

$t=1$; $\ln(d)_{merror} = 1.609438 - 6.156554 = -4.547116$

$t=2$; $\ln(d)_{merror} = 2.944439 - 6.156554 = -3.212115$

$t=3$; $\ln(d)_{merror} = 3.912023 - 6.156554 = -2.244531$

$t=4$; $\ln(d)_{merror} = 5.017280 - 6.156554 = -1.139274$

$t=5$; $\ln(d)_{merror} = 6.152733 - 6.156554 = -0.003821$

$t=6$; $\ln(d)_{merror} = 7.268920 - 6.156554 = 1.112366$

$t=7$; $\ln(d)_{merror} = 8.414496 - 6.156554 = 2.257942$

$t=8$; $\ln(d)_{merror} = 9.467769 - 6.156554 = 3.311216$

$t=9$; $\ln(d)_{merror} = 10.62437 - 6.156554 = 4.467818$

$t=10$; $\ln(d)_{merror} = 11.61747 - 6.156554 = 5.460921$

$$R = \sqrt{\frac{\sum(y_{sim} - y_{mean})^2}{\sum(y - y_{mean})^2}}$$

where $y_{sim} = \ln(d)_{sim}$

$y_{mean} = \ln(d)_{mean}$

$y = \ln(d)$

$\sum(y_{sim} - y_{mean})^2 = \ln(d)_{sim\,error}$

$\sum(y - y_{mean})^2 = \ln(d)_{merror}$

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ENG 382 - Engineering mathematics IV

Assignment 6

$$\sum (y_{sim} - y_{mean})^2 = 30.504512 + 19.522886 + 10.981622 + 4.880719 + 1.220181 + 0 + 1.220181 + 4.880719 + 10.981622 + 19.522886 + 30.504512$$

$$\sum (y_{sim} - y_{mean})^2 = 134.21984$$

$$\sum (y - y_{mean})^2 = 29.848816 + 20.676264 + 10.317683 + 5.039919 + 1.297945 + 0.00046 + 1.237358 + 5.098302 + 10.964151 + 19.961398 + 29.821658$$

$$\sum (y - y_{mean})^2 = 134.26154$$

$$\therefore R = \sqrt{\frac{134.21984}{134.26154}}$$

$$R = 0.999844694$$

$$R^2 = (0.999844694)^2$$

$$R^2 = 0.999689412 2$$

BULLEM, FLORENCE ILUEH-OCHUWEH

16/ENG01/005

CHEMICAL ENGINEERING

ENG 382- ENGINEERING MATHEMATICS IV

SOLUTION FROM MICROSOFT EXCEL

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.99984483
R Square	0.99968969
Adjusted R Square	0.99965521
Standard Error	0.06803826
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	134.219833	134.219833	28994.1419	4.2259E-17
Residual	9	0.04166285	0.00462921		
Total	10	134.261496			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.63346515	0.03837876	16.5056196	4.9026E-08	0.54664637	0.720284	0.546646	0.720284
X Variable 1	1.10461772	0.00648719	170.276663	4.2259E-17	1.08994266	1.119293	1.089943	1.119293

For calculating Ln(B)

=Ln(B2); the formula is applied downwards

t(hr)	d(m)	ln(d)	ln(d)sim	ln(d)simerror	ln(d)merror
0	2	0.693147	0.633465	-5.523089	-5.463407
1	5	1.609438	1.738083	-4.418471	-4.547116
2	19	2.944439	2.842701	-3.313853	-3.212115
3	50	3.912023	3.947318	-2.209235	-2.244531
4	151	5.01728	5.051936	-1.104618	-1.139274
5	470	6.152733	6.156554	0.000000	-0.003821
6	1435	7.26892	7.261171	1.104618	1.112366
7	4512	8.414496	8.365789	2.209235	2.257942
8	12936	9.467769	9.470407	3.313853	3.311216
9	41125	10.62437	10.57502	4.418471	4.467818
10	111021	11.61747	11.67964	5.523089	5.460921

dmean	ln(d)mean	sum	sum
15611.45	6.156554	134.2198329	134.261496

R	Rsquared
0.99984483	0.99968969

Calculating ln(d)mean

=AVERAGE(C2:C12)

Calculating ln(d)sim

=K21+ (A2*K22); K21 and K22 has been made constant, therefore the formula is applied downwards with only t (A column) changing.

Where K21 = intercept (from regression analysis)

K22 = X-variable (from regression analysis)

Calculating ln(d)simerror

=D2-C15 (ln(d)sim - ln(d)mean)

C15 (ln(d)mean) has been kept constant, hence the formula is applied downwards.

Calculating ln(d)merror

=C2-C15 (ln(d) - ln(d)mean)

C15 (ln(d)mean) has been kept constant, hence the formula is applied downwards.

The sum of the squares is needed;

Hence, $\sum \ln(d)\, simerror$ = SUMSQ(E2:E12)

$\sum \ln(d)\, merror$ = SUMSQ(F2:F12)

R = $\sqrt{\frac{(y_{sim}-y_{mean})^2}{(y-y_{mean})^2}}$

R = $\sqrt{\frac{(\ln(d)_{sim}-\ln(d)_{mean})^2}{(\ln(d)-\ln(d)_{mean})^2}}$

R = $\sqrt{\frac{(\ln(d)simerror)^2}{(\ln(d)merror)^2}}$

R = SQRT (E15/F15)

Where E15 = $\sum \ln(d)\, simerror$

F15 = $\sum \ln(d)\, merror$

R = 0.99984483

Rsquare =E18^2

Where E18 = R

Rsquare = 0.99968969

ENG382_ASSIGNMENT6.xlsx - Microsoft Excel

t(hr)	d(m)	ln(d)	ln(d)sim	ln(d)simerror	ln(d)merror
0	2	0.693147	0.633465	-5.523089	-5.463407
1	5	1.609438	1.738083	-4.418471	-4.547116
2	19	2.944439	2.842701	-3.313853	-3.212115
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7	4512	8.414496	8.365789	2.209235	2.257942
8	12936	9.467769	9.470407	3.313853	3.311216
9	41125	10.62437	10.57502	4.418471	4.467818
10	111021	11.61747	11.67964	5.523089	5.460921

dmean	ln(d)mean	sum	sum
15611.45	6.156554	134.2198329	134.261496

R	Rsquared
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Residual	9	0.04166285	0.00462921		
Total	10	134.261496			

	Coefficients	*tandard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95*
Intercept	0.63346515	0.03837876	16.5056196	4.9026E-08	0.54664637	0.720284	0.546646	0.7202
X Variable 1	1.10461772	0.00648719	170.276663	4.2259E-17	1.08994266	1.119293	1.089943	1.1192

SOLUTION FROM MATLAB

```
commandwindow
clear
clc
collins = xlsread('ENG382_ASSIGNMENT6_2');
t = collins(:,1)
d = collins(:,2)
d = log(d)
[xr xc] = size(t)
t0 = ones(xr,1)
t1 = [t0 t]
maxwell = regress(d,t1)
lnalpha = maxwell(1)
lnbeta = maxwell(2)
alpha = exp(maxwell(1))
beta = exp(maxwell(2))
lnd = lnalpha + lnbeta*t
Rvalue = corr(d,lnd)
Rsquare = Rvalue^2
```

Results

```
collins =

           0           2
           1           5
           2          19
           3          50
           4         151
           5         470
           6        1435
           7        4512
           8       12936
           9       41125
          10      111021


t =

     0
     1
     2
```

```
     3
     4
     5
     6
     7
     8
     9
    10

d =
           2
           5
          19
          50
         151
         470
        1435
        4512
       12936
       41125
      111021

d =
      0.69315
       1.6094
       2.9444
        3.912
       5.0173
       6.1527
       7.2689
       8.4145
       9.4678
```

```
       10.624
       11.617

xr =
    11

xc =
     1

t0 =
     1
     1
     1
     1
     1
     1
     1
     1
     1
     1
     1

t1 =
     1     0
     1     1
     1     2
     1     3
     1     4
     1     5
     1     6
     1     7
     1     8
```

```
     1     9
     1    10

maxwell =
      0.63347
       1.1046

lnalpha =
      0.63347

lnbeta =
       1.1046

alpha =
       1.8841

beta =
       3.0181

lnd =
      0.63347
       1.7381
       2.8427
       3.9473
       5.0519
       6.1566
       7.2612
       8.3658
       9.4704
       10.575
        11.68
```

```
Rvalue =
      0.99984

Rsquare =
      0.99969
```

Comment on Rsquare Values

The correlation coefficient (Rsquare) is greater than 0.80(80%), it is 0.9969 which is equivalent to 99.69%. This shows how efficient the model is, hence it can be used for solving further problems.