DAFE MERCY EBELE
16/ENG04/014
Elect/Elect Engineering
ENG382

Question

If the dynamics of a crude oil spreading system is described by the expression given in Equation (1) where d is the distance of spread, α & β are model constants, and the time series data generated from the experiments carried out on the system are as given in Table 1, ~~esta~~ estimate the values of α and β

a.) manually
b.) with the aid of Microsoft Excel using the regression tool,
c.) With the aid of MATLAB using the regress command and
d.) In each of the cases (a) to (c) estimate the values of the
i) Correlation Coefficient (R)
ii) square of the Correlation Coefficient (R^2) and
e) Comment on the results obtained in (d)

$d = \alpha\beta^t$

Table 1

t (hr)	d (m)
0	2
1	5
2	19
3	50
4	151
5	470
6	1435
7	4512

8	12936
9	41125
10	111021

Answer

$d = \alpha \beta^t$

$\log d = \log \alpha + \log \beta^t$ (Taking the log of both sides)

$\log d = \log \alpha + t \log \beta$

Where

$Y = a_0 + a_1 x$

$a = \log \beta$, $x = t$, $a_0 = \log \alpha$, $Y = \log d$

$Yx = a_0 x + a_1 x^2$

$\Sigma Y = N\alpha_0 + a_1 \Sigma x$ --(1)

$\Sigma Yx = a_0 \Sigma x + a_1 \Sigma x^2$ -(2)

where $N = 11$

from the table below calculated

$\Sigma Y =$ ~~29.41~~ 29.4113

$\Sigma x = 55$

$\Sigma Yx = 199.8269$

$\Sigma Y^2 = 103.962$

$\Sigma x^2 = 385$

t(x)	d(m)	log d(y)	yx	x^2	y^2
0	2	0.30103	0	0	0.090619
1	5	0.69597	0.69897	1	0.488559
2	19	1.278754	2.552507	4	1.635211
3	50	1.69897	5.09091	9	2.886499
4	151	2.178977	8.715908	16	4.747941
5	470	2.672098	13.36049	25	7.140107
6	1435	3.156852	18.94111	36	9.965714
7	4512	3.654369	25.58058	49	13.35441
8	12936	4.1118	32.8944	64	16.9069
9	41125	4.614106	41.52695	81	21.28997
10	111021	5.045405	50.45405	100	25.46511
Σ 55	171726	29.41133	199.8269	385	103.962

$29.41133 = 11a_0 + 55a_1$ --- (i)

$199.8269 = 55a_0 + 385a_1$ --- (ii)

making a_0 subject of the formula

$29.41133 - 55a = 11a_0$

$a_0 = \frac{29.41133 - 55a_1}{11}$ --- (iii)

Sub (iii) into (ii)

$199.8269 = \cancel{55}^{5}\left[\frac{29.41133 - 55a_1}{\cancel{11}}\right] + 385a_1$

$199.8269 = 5(29.41133 - 55a_1) + 385a_1$

$199.8269 = 147.05665 - 275a_1 + 385a_1$

$199.8269 - 147.05665 = 385a_1 - 275a_1$

$52.77025 = 110a_1$

$a_1 = \frac{52.77025}{110}$

$a_1 = 0.4797$

$a_0 = \frac{29.41133 - 55 \times 0.4297}{11}$

$$a_0 = \frac{29.41133 - 26.3835}{11} = a_0 = 0.2752$$

$$a_0 = \log \alpha$$
$$\alpha = \log^{-1} 0.2752$$
$$\alpha = 1.8845$$

$$a_1 = \log \beta$$
$$\beta = \log^{-1} 0.4797$$
$$\beta = 3.0178$$

Corelation Coefficient (R)

$$R = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{(N\Sigma x^2 - (\Sigma x)^2 (N\Sigma y^2 - (\Sigma y)^2)}}$$

$$R = \frac{(11 \times 199.8569) - (55 \times 29.41133)}{\sqrt{((11 \times 385) - 55^2)(11 \times 103.962 - (29.41133)^2)}}$$

$$R = 0.9998460887$$

$$R^2 = (0.9998460887)^2$$
$$= 0.9996922011$$

$$R = 0.9998$$

$$R^2 = 0.9997$$

e.) From the answer above, it shows that $R^2 < R$