

ASSIGNMENT VI

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COMPUTER ENGINEERING

QUESTION

If the dynamics of a Crude oil Spreading system is described by the expression given in Eqn(1), where d is the distance of spread, α and β are Model constants and the time series data generated from the experiments carried out in the system are g as given in Table 1, estimate the values of α and β ,

- Manually
- With the aid of Microsoft excel using the regression tool,
- With the aid of MATLAB using the regress Command, and
- In each of the Cases (a) to (c), estimate the values of the
 - Correlation Coefficient (R)
 - Square of the Correlation Coefficient (R^2) and
- Comment on the results obtained in (d)

$$d = \alpha \beta^2 \quad \dots (1)$$

Solution

a) Eqn (1) is non linear

finding log of both sides

$$\log d = \log x + \log B^t$$

$$\log d = \log x + t \log B$$

Comparing $y = mx + c$

$$y = \log d; c = \log x; x = t; m = \log B$$

Standard equation: $y = a_0 + a_1 x$

$$a_0 = c \quad \& \quad a_1 = m$$

S/N	x/t (cm)	d (cm)	y/log d	x y	x ²	y ²
1	0	2	0.30102996	0	0	0.090619058
2	1	5	0.698970004	0.69897	1	0.488559067
3	2	19	1.278753601	2.557507	4	1.635210772
4	3	50	1.698970004	5.096910013	9	2.886499076
5	4	151	2.178976947	8.715908	16	4.747940537
6	5	470	2.672097858	13.36049	25	7.140106962
7	6	1435	3.156851901	18.94111	36	9.965715925
8	7	4512	3.654369091	25.58058	49	13.35441345
9	8	12936	4.111800007	32.8944	64	16.9068993
10	9	41125	4.614105911	41.52695	81	21.28997536
11	10	111021	5.045405185	50.45405	100	25.45611297
$\Sigma =$	55		29.41183046	199.826885	385	103.9620485

$$y = a_0 + a_1 x$$

$$\Sigma y = a_0 N + a_1 \Sigma x$$

$$\Sigma xy = a_0 \Sigma x + a_1 \Sigma x^2$$

from the table

$$29.41133046 = a_0 + 11 + a_1 + 55 \quad \text{--- (i)}$$

$$199.8268839 = a_0 + 55 + a_1 + 385 \quad \text{--- (ii)}$$

Using elimination

$$\text{eqn (i)} \times 5 \quad \text{---} \quad \text{eqn (ii)} \times 1$$

$$147.0566523 = 55a_0 + 275a_1$$

$$199.8268839 = 55a_0 + 385a_1$$

Subtracting,

$$52.7702316 = 110a_1$$

$$a_1 = 0.479729379 //$$

$$\text{eqn (i)} \times 7 \quad \text{---} \quad \text{eqn (ii)} \times 1$$

$$205.8793132 = 77a_0 + 385a_1$$

$$199.8268839 = 55a_0 + 385a_1$$

Subtracting,

$$6.052429243 = 22a_0$$

$$a_0 = 0.27511042 //$$

$$\therefore y = 0.27511042 + 0.479729379x //$$

Recall

$$q_0 = C_2 \log x$$

$$x = 10^{q_0}$$

$$x = 10^{0.275110112}$$

$$x = 1.884128072 \approx 1.88 //$$

$$q_1, ZM = \log \beta$$

$$\beta = 10^{q_1}$$

$$\beta = 10^{0.479729379}$$

$$\beta = 3.018070489 \approx 3.02 //$$

$$\therefore x = 1.88 \quad \& \quad \beta = 3.02$$

di) Correlation Coefficient, R

$$R = \frac{N \sum xy - [\sum x][\sum y]}{\sqrt{[N \sum x^2 - (\sum x)^2][N \sum y^2 - (\sum y)^2]}}$$

$$R = \frac{[11 \times 199.8268839] - [55 \times 29.41153046]}{\sqrt{[(11 \times 385) - (55)^2] * [(11 \times 103.9620485) - (29.41153046)^2]}}$$

$$R = 0.999844832,$$

ii) Square of the Correlation Coefficient R^2

$$R^2 = (0.999844832)^2$$

$$R^2 = 0.999689689 //$$

e) The values for the Correlation Coefficient obtained falls between the standard range for Correlation, 0.8 - 1.0

$$\text{AS } 0.8 < 0.999844832 < 1.0$$

Therefore the Variables $x(t)$ and $y(\log d)$ Correlate.