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DEPARTMENT: MECHANICAL ENGINEERING

If it is discovered that 600 ft^3/min of fresh air flows into a room containing 20,000 ft^3 litre of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of 600 ft^3/min. If the room contains no fresh air initially.

a Develop a model for the amount of fresh air in the room at any time t

Solution

Let y(t) be the amount of air at any time (t) in ft^3 in the room

$\frac{dy}{dt}$ ⇒ Fresh air inflow rate − Fresh air outflow rate

$\frac{dy}{dt}$ ⇒ change in flow rate

Fresh air inflow ⇒ 600 ft^3/min

$\therefore \frac{600}{20,000} = 0.03$ min

i.e 0.03 of y(t) is the outflow = $0.03y$ ft^3/min

$\therefore \frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03[y - 20,000]$

Thus equation can be seperated and integrated;

$\frac{dy}{(y - 20,000)} = -0.03dt$

Find the integral of both sides

$\ln(y - 20,000) = -0.03t + c$

$y - 20,000 = e^{(-0.03t + c)}$

$y - 20,000 = e^{-0.03t} \cdot e^c$

Recall $C = e^c$ = Natural equation

$\therefore y - 20,000 = e^{-0.03t} \cdot C$ ... (1)

At $t = 0$, $y(t) = 0$ Since the room contain no fresh air initially

put $y=0$, $t=0$ into eqn (1)

$y = 20,000 = e^{-0.03t} \cdot C$

$0 - 20,000 = e^{-0.03(0)} \cdot C$

$-20,000 = e^{0} \cdot C$

$C = -20,000$ --- (2)

put eqn (2) into (1)

$y = 20,000 - 20,000^{-0.03t}$

$y = 20,000(1 - e^{-0.03t})$ --- (3)

Equation (3) is the model for the amount of fresh air in the room

b Calculate the time at which 90% of the air in the room will become fresh

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20,000$ ie 90% of air in the room

$= 18,000 ft^3$

$y = 20,000(1 - e^{-0.03t})$

$18000 = 20,000[1 - e^{-0.03t}]$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = 77$ mins

c With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t=0$ to $t=6$ hours using a step of 5 mins.

- Command window
- clear
- cle
- close all
- syms y,t
- y = 20000 * [1 - exp(-0.03 * t)]

- t = 0:5:360
- yn = subs(y)
- plot (t, yn)
- x label ('Time (min)')
- y label ('Flow rate of fresh air (ft^3/min)')
- Grid on
- Grid minor
- Axis tight

Output: [MATLAB DISPLAY]

d Determine the steady state value of the amount of fresh air in the room.

Solution

The steady state value is 20,000 ft³/ at 215 mins (3hrs and 35mins) of exponential approach

e Comment:

The functions above shows an exponential equation to the curve of 20,000 ft³ as y increases with time. Also, when the steady state value approaches 20,000 ft³ at 215 mins and continues till 360 min (6hrs). The model derived becomes more real as gas (pneumatic technology), although rough because mixing may be imperfect.