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Que 1

If the dynamics of a crude oil spreading system is described by the expression given in equ (1) where d is the distance of spread, α and β are Model Constants, and the time series data generated from the experiments carried out on the system are as given in table 1, estimate the values of α and β,

(a) manually
(b) with the aid of Microsoft Excel using the regression tool
(c) with the aid of MATLAB using the regress command, and
(d) in each of the cases (a) to (c), estimate the values of the
(i) Correlation Coefficient (R),
(ii) Square of the Correlation Coefficient (R^2), and
(e) Comment on the results obtained in (d)

$$d = \alpha t^{\beta} \quad (1)$$

Table 1: Time series data of the System

t(hr)	d(m)
0	2
1	5
2	8
3	50
4	151

t(hr)	d(m)
5	470
6	1435
7	4512
8	12936
9	41125
10	111021

Solution

$d = \alpha \beta^t$

taking the log of both sides

$\log d = \log \alpha \beta^t$

$\log d = \log \alpha + \log \beta^t$

$\log d = t \log \beta + \log \alpha$

Comparing the expression above with that of the Equation of a straigh line

$y = a_0 + a_1 x$

$a_0 = \log \alpha$; $a_1 = \log \beta$; $y = \log d$; $x = t$

$xy = a_0 x + a_1 x^2$

$\sum y = a_0 N + \sum a_1 x$

$\sum xy = \sum a_0 x + \sum a_1 x^2$

in Matrix form

$$\begin{bmatrix} N & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}$$

N	x	x^2	xy	y
1	0	0	0	0.3010299996
2	1	1	0.6989700004	0.6989700004
3	2	4	2.557507202	1.278753601
4	3	9	5.096910013	1.6989700004
5	4	16	8.715907789	2.1789769947
6	5	25	13.36048929	2.672097858
7	6	36	18.94111141	3.156851901
8	7	49	25.58058364	3.654369091

N	x	x^2	xy	y
9	8	64	32.8944006	4.11180007
10	9	81	41.5269532	4.61416591
11	10	100	50.45405135	5.04540535

$\Sigma y = 29.41133046$

$\Sigma x = 55$

$\Sigma xy = 119.8268839$

$\Sigma x^2 = 385$

$$a_0 = \frac{\begin{bmatrix} \Sigma y & \Sigma x \\ \Sigma xy & \Sigma x^2 \end{bmatrix}}{\begin{bmatrix} N & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix}}$$

$$a_0 = \frac{(29.41133046)(385) - (55)(119.8268839)}{(11 \times 385) - (55 \times 55)}$$

$a_0 = 0.27511$

$$a_1 = \frac{\begin{bmatrix} N & \Sigma y \\ \Sigma x & \Sigma xy \end{bmatrix}}{\begin{bmatrix} N & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix}}$$

$$a_1 = \frac{(11 \times 119.8268839) - (29.41133046 \times 55)}{(11 \times 385) - (55 \times 55)}$$

$a_1 = 0.47973$

$a_0 = \log \alpha \qquad a_1 = \log \beta$

$\alpha = 10^{a_0} \qquad \beta = 10^{a_1}$

$\alpha = 1.88341, \quad \beta = 3.01881$

(di) Correlation Coefficient (R)

$$R = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{(N\Sigma x^2 - (\Sigma x)^2)(N\Sigma y^2 - (\Sigma y)^2)}}$$

$$R = \frac{(11 \times 199.8268839) - (55)(29.411133046)}{\left((11 \times 385 - 55^2) * (11 \times 103.9420485 - (29.411133046)^2)\right)^{1/2}}$$

(for Manual Method)

$R = 0.999844831$2

$R(\text{square}) = (0.999844831)^2$

$= 0.9996896865$

(for MATLAB)

$R = 0.9998$; $R(\text{square}) = 0.9997$

(for Excel)

$R = 0.999845$

$R(\text{square}) = 0.999689$

(d) It was observed from the various methods used to calculate for the Correlation Coefficient (R) and its square that R(square) < R. Also it was observed that the various answers gotten were approximately equal.