

$$S(100) = \frac{1}{100} \left(\frac{100}{100} \right) = 1.0$$

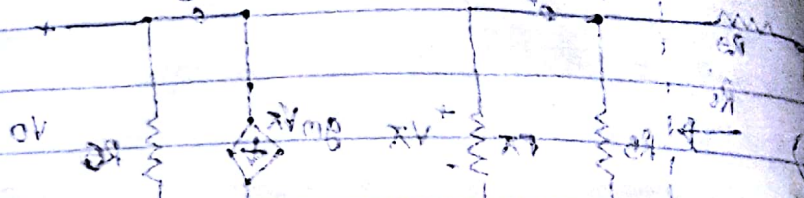
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COURSE: Engineering Maths

$$C(100) = \frac{1}{100} \left(\frac{100}{100} \right) = 1.0$$



Que. (4+10+10+10)

If the dynamics of a crude oil spreading system is described by the expression given in equ (1), where d is the distance of spread, α and β are Model constants, and the time series data generated from the experiments carried out on the system are as given in table 1, estimate the values of α and β .

- manually
- with the aid of Microsoft Excel using the regression tool
- with the aid of MATLAB using the regress commands, and
- In each of the cases (a) to (c), estimate the values of
 - Correlation Coefficient (R),
 - Square of the Correlation Coefficient (R^2), and
 - Comments on the results obtained in (i)

Table 1 Time series data of the system

P.02- (7.6HR) = 0V	VA	d(m)	5.0 = 100.00V, 2.0 = 8.0
0.00	0.8	2	0.1005 = 1.8, 0.1005 = 8
0.01	0.8	5	0.008 = 0.1, 0.008 = 0.1
0.02	0.8	10	0.008 = 0.1, 0.008 = 0.1
0.03	0.8	15	0.008 = 0.1, 0.008 = 0.1

$t(\text{hr})$	$d(\text{cm})$
5	470
6	1435
7	4512
8	12936
9	41125
10	111021

Solution -

$$d = \alpha \beta^t$$

taking the log of both sides

$$\log d = \log \alpha \beta^t$$

$$\log d = \log \alpha + \log \beta^t$$

$$\log d = t \log \beta + \log \alpha$$

Comparing the expression above with that of the equation of a straight line

$$y = a_0 + a_1 x$$

$$a_0 = \log \alpha; a_1 = \log \beta; y = \log d; x = t$$

$$xy = a_0 x + a_1 x^2$$

$$\sum y = a_0 N + \sum a_1 x$$

$$\sum xy = \sum a_0 x + \sum a_1 x^2$$

in Matrix form

$$\begin{bmatrix} N & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}$$

N	x	x ²	xy	y
1	0	0	0	0.301029996
2	1	1	0.698970004	0.698970004
3	2	4	2.557507202	1.278753601
4	3	9	5.096910013	1.698970004
5	4	16	8.715907789	2.178976947
6	5	25	13.36049929	2.672097858
7	6	36	18.94111141	3.1568519091
8	7	49	25.58058364	3.654369091

N	x	x ²	xy	y
9	8	64	32.8944006	4.111800007
10	9	81	41.5269532	4.614165911
11	10	100	50.4540535	5.045405935

$$(100)(8.111800007) - (32.8944006)(11) = 1$$

$$\sum y = 29.41133046$$

$$\sum x = 55$$

$$\sum xy = 119.8268839$$

$$\sum x^2 = 385$$

$$\sum y^2 = 15.96111111$$

$$a_0 = \frac{\sum y \sum x - (\sum xy)}{\sum x^2 - (\sum x)^2}$$

$$a_0 = \frac{(29.41133046)(55) - (119.8268839)}{(385) - (55 \times 55)}$$

$$a_0 = \frac{(1617.623274) - (119.8268839)}{(385) - (3025)}$$

$$a_0 = \frac{1497.79639}{-2640}$$

$$a_0 = -0.567347$$

$$a_1 = \frac{\sum y^2 - \frac{(\sum y)^2}{N}}{\sum x^2 - \frac{(\sum x)^2}{N}}$$

$$a_1 = \frac{15.96111111 - \frac{(29.41133046)^2}{11}}{385 - \frac{(55)^2}{11}}$$

$$a_1 = \frac{15.96111111 - 78.11111111}{385 - 275}$$

$$a_1 = \frac{-62.15}{110}$$

$$a_1 = -0.565$$

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⑥ Correlation Coefficient (R)

$$R = \frac{N \sum xy - (\sum x)(\sum y)}{\sqrt{(N \sum x^2 - (\sum x)^2)(N \sum y^2 - (\sum y)^2)}}$$

$$R = \frac{(11 \times 199.8268839) - (55)(29.41133046)}{\sqrt{(11 \times 385 - 55^2) * (11 \times 103.9420485 - (29.41133046)^2)}}^{1/2}$$

(for Manual Method)

$$R = 0.9998448312$$

$$R_{\text{square}} = (0.9998448312)^2 = 0.9996896865$$

(for MATLAB)

$$R = 0.9998, R_{\text{square}} = 0.9997$$

(for Excel)

$$R = 0.9998448312$$

$$R_{\text{square}} = 0.9996896865$$

⑦ It was observed from the various methods used to calculate for the Correlation Coefficient (R) and its square that $R_{\text{square}} < R$. Also it was observed that ~~the~~ ^{various} the answers gotten were approximately equal.